L. M. MILNE-THOMSON

PROFESSOR OF MATHEMATICS IN THE ROYAL NAVAL COLLEGE
GRESHAM PROFESSOR OF GEOMETRY

Jacobian Elliptic Function Tables

A GUIDE TO PRACTICAL COMPUTATION WITH ELLIPTIC FUNCTIONS AND INTEGRALS TOGETHER WITH TABLES OF
sn u, cn u, dn u, $Z(u)$

DOVER PUBLICATIONS INC.

1950

THE DOVER SERIES IN
MATHEMATICS AND PHYSICS

W. PRAGER, *Consulting Editor*

Printed & Bound in the United States of America

"Ante biennium fere, cum theoriam functionum ellipticarum accuratius examinare placuit, incidi in quaestiones quasdam gravissimas, quae et theoriae illi novam faciem creare, et universam artem analyticam insigniter promovere videbantur."

CAROLUS GUSTAVUS JACOBUS JACOBI, 1829

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GRAPHS AND TABLES

Preface

THE WIDESPREAD BELIEF that calculations involving elliptic functions are difficult is due not to the nature of the calculations themselves but to the lack of suitable numerical tables wherewith to perform them. Calculations involving sines and cosines would present analogous difficulties in the absence of tables of trigonometric functions. Elliptic functions are but natural generalisations of trigonometric functions on the one hand and hyperbolic functions on the other.

Elliptic functions arise in every branch of applied science. To give an exhaustive list would be impossible but a few instances will make the point. The pendulum performing oscillations of finite amplitude or making complete revolutions (see p. 38); Euler's equations of motion of a rigid body (see p. 22); resistance of projectiles; capillary phenomena; the potential of a gravitating or electrified ellipsoid; the mutual inductance of circular currents; the bending of an elastic rod; an ellipsoid moving through fluid; the flow of a viscous fluid in a convergent or divergent channel; wind tunnel interference*; conformal transformation (see p. 33).

The need for interpolable tables of elliptic functions is therefore clear. The first systematic tables of Jacobian functions ever to be published appeared long since† and the present book is the natural successor to that pioneer volume.

The problem of the scientist who wishes to perform a numerical calculation is to find a number. In so far as elliptic functions are concerned the present tables are so arranged that the number

*Milne-Thomson, *Theoretical Aerodynamics*, London, (1948).
†Milne-Thomson, *Die elliptischen Funktionen von Jacobi*, Berlin, (1931).

may be found with simplicity, directness, and speed. For each value of the parameter m (i.e. the squared modulus) the values of the three Jacobian elliptic functions, sn u, cn u, dn u, are arranged in columns for values of the argument u progressing by the interval 0·01 up to a value in excess of the corresponding quarter period K, which is printed at the foot of each column. The values of these functions are all five-figure decimal numbers so that the decimal point is unnecessary and is omitted. To make interpolation simple for intermediate values of u printed differences are given. The section on Numerical examples, p. 6, explains in detail the method of using the tables.

The limitation of the tabular values of sn u, cn u, dn u to five decimal places has the advantage of rendering interpolation easier, and of allowing compression of the tables without sacrifice of efficiency, since five-figure accuracy suffices for all but the most exceptional calculations. Should an isolated value be required to a larger number of figures, this can be found from the trigonometric series on p. 13.

It is worth noting that when $m = 0$, sn u = sin u, cn u = cos u and therefore the columns headed 0·0 provide tables of these trigonometric functions. Again when $m = 1$, sn u = tanh u, cn u = sech u so that the columns headed 1·0 provide a table of these hyperbolic functions.

While the three Jacobian functions suffice for calculation of elliptic integrals of the first kind, the inclusion of a table of Jacobi's zeta function $Z(u)$ enables elliptic integrals of the second and third kinds to be evaluated. This table is here given to seven decimal places as originally published.*

The tables of the complete elliptic integrals K, K', E, E' and the nome q are adapted from ten-figure tables of these functions.† They are here given to eight figures to allow scope for special calculations. It is interesting to note how the adoption of the parameter m and its complement $m_1 = 1 - m$ allows compression without sacrifice of clarity.

*Milne-Thomson, *Proc. Royal Soc.*, Edinburgh, 52. (1932) pp. 236–250.
†Milne-Thomson, *Proc. London Math. Soc.* (2) 33. (1932) p. 162;
Journ. London Math. Soc. 5 (1930) p. 148.

A comprehensive collection of formulae is included. These have been carefully chosen with a view to facilitating calculations which may present themselves. Thus, for example, we can evaluate Weierstrass's $\wp$-function as well as the Jacobian elliptic functions for complex values of the argument, and for all real values positive or negative of the parameter. Negative values of the parameter correspond to a purely imaginary modulus.

The inclusion of formulae special to particular branches of knowledge would have been not only invidious but also impracticably extensive. An exception has been made in favour of some conformal transformations which cover ground common to several sciences.

This small volume is intended as a practical tool for the user of elliptic functions. Those who wish to learn about the theoretical aspect could not do better than read Neville's* attractive account of the theory.

In conclusion, I wish to thank Dover Publications for the care and attention which they have given to the production which is of major importance in the presentation of mathematical tabular matter.

L. M. MILNE-THOMSON

Royal Naval College
Greenwich

November 1949

*E. H. Neville, *Jacobian Elliptic Functions*, Oxford, 1944.

The Jacobian Elliptic Functions

We begin by noting the equivalence of the integrals

$$u = \int_0^x \frac{dt}{[(1 - t^2)(1 - mt^2)]^{1/2}} = \int_0^\varphi \frac{d\theta}{(1 - m \sin^2 \theta)^{1/2}}$$

which are related by the substitutions

$$t = \sin \theta, \qquad x = \sin \varphi.$$

The *Jacobian elliptic functions* tabulated in this book may be defined by the relations

$$\text{sn}\, u = \sin \varphi, \qquad \text{cn}\, u = \cos \varphi, \qquad \text{dn}\, u = (1 - m \sin^2 \varphi)^{1/2}$$

or by the equivalent set

$$\text{sn}\, u = x, \qquad \text{cn}\, u = (1 - x^2)^{1/2}, \qquad \text{dn}\, u = (1 - mx^2)^{1/2},$$

where the positive square root is to be taken in every case.

The number u will be called the *argument* and the number m the *parameter* of the functions.

The *complementary parameter* is the number

$$m_1 = 1 - m$$

When it is required to call specific attention to the dependence of the functions on the parameter, instead of sn u, cn u, dn u we write

$$\text{sn}\,(u \mid m), \qquad \text{cn}\,(u \mid m), \qquad \text{dn}\,(u \mid m)$$

with a vertical stroke separating argument and parameter.

At this stage we note in passing that it is usual to regard the

Jacobian elliptic functions as dependent on the *modulus* k, where $k^2 = m$, and in this notation they would be written

$$\text{sn}\,(u, k), \qquad \text{cn}\,(u, k), \qquad \text{dn}\,(u, k)$$

with a comma separating argument and modulus. We also observe that when the parameter is negative, the modulus is imaginary. The *complementary modulus* is $k' = (1 - k^2)^{1/2}$.

Since in applications it is the parameter rather than the modulus which is given, we shall adopt the notation $\text{sn}\,(u \mid m)$.

The three Jacobian elliptic functions are one-valued functions of the argument u and are doubly periodic. The numbers K and iK' given by

$$K = K(m) = \int_0^{\pi/2} \frac{d\theta}{(1 - m \sin^2 \theta)^{1/2}},$$

$$iK' = iK'(m) = i \int_0^{\pi/2} \frac{d\theta}{(1 - m_1 \sin^2 \theta)^{1/2}}$$

are the real and imaginary quarter periods. In fact it can be proved that if $\text{pq}\,u$ is any one of the Jacobian elliptic functions, then

$$\text{pq}\,(u + 4K \mid m) = \text{pq}\,(u + 4iK' \mid m) = \text{pq}\,(u \mid m)$$

so that $4K$ and $4iK'$ are periods.

It is clear from their definitions that $K'(m)$ is the same function of m_1 as $K(m)$ is of m, and therefore that

$$K(m) = K'(1 - m).$$

When $m = 0$, we have $K(0) = \frac{1}{2}\pi$, $K'(0) = \infty$, while $\varphi = u$ so that

$$\text{sn}\,(u \mid 0) = \sin u, \qquad \text{cn}\,(u \mid 0) = \cos u, \qquad \text{dn}\,(u \mid 0) = 1$$

Thus when $m = 0$ the elliptic functions degenerate into circular functions with quarter period $\frac{1}{2}\pi$ and period 2π.

Similarly when $m = 1$, we have $K(1) = \infty$, $iK'(1) = \frac{1}{2}i\pi$, while $\sin \varphi = x = \tanh u$ so that

$$\text{sn}\,(u \mid 1) = \tanh u, \qquad \text{cn}\,(u \mid 1) = \text{sech}\, u = \text{dn}\,(u \mid 1)$$

and the elliptic functions degenerate into hyperbolic functions with quarter period $\frac{1}{2}i\pi$ and period $2i\pi$.

Just as in trigonometry in addition to the functions sin u and cos u it is usual to consider their reciprocals cosec u, sec u and their ratios tan u, cot u, so with the elliptic functions we consider their reciprocals and ratios written

$$\operatorname{ns} u = \frac{1}{\operatorname{sn} u}, \qquad \operatorname{nc} u = \frac{1}{\operatorname{cn} u}, \qquad \operatorname{nd} u = \frac{1}{\operatorname{dn} u}$$

$$\operatorname{sc} u = \frac{\operatorname{sn} u}{\operatorname{cn} u}, \qquad \operatorname{cs} u = \frac{\operatorname{cn} u}{\operatorname{sn} u}, \qquad \operatorname{ds} u = \frac{\operatorname{dn} u}{\operatorname{sn} u}$$

$$\operatorname{sd} u = \frac{\operatorname{sn} u}{\operatorname{dn} u}, \qquad \operatorname{dc} u = \frac{\operatorname{dn} u}{\operatorname{cn} u}, \qquad \operatorname{cd} u = \frac{\operatorname{cn} u}{\operatorname{dn} u}$$

comprising, in all, 12 Jacobian elliptic functions.

Again the *inverse* functions obtained as solutions of equations such as sn $u = a$, cd $u = b$ are written $u = \operatorname{sn}^{-1} a$, $u = \operatorname{cd}^{-1} b$, and so on.

The above definitions suffice for the numerical applications of the functions, but a better insight into their structure is attained by considering them as functions of the complex variable $z = x + iy$. The following brief outline offers an alternative definition, to which the foregoing can be shown to be completely equivalent.

The Jacobian elliptic functions are 12 in number and may be readily defined with respect to the following doubly infinite rectangular array of lattice points.

$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$
$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$
$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$	$\cdot c$	$\cdot s$
$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$	$\cdot d$	$\cdot n$

The pattern is repeated indefinitely on all sides. If we denote by the (complex) number K_s an arbitrary point labelled s, we thereby define adjacent points labelled c, n, d situated respectively east, north, and southwest from s. We denote these points by the complex numbers K_c, K_n, K_d. If we take K_s as origin, it is plain that $K_s = 0$ and that

$$K_s + K_c + K_d + K_n = 0.$$

We shall suppose that the scale of the above lattice is such that, with properly chosen axes of reference,

$$K_c = K, \qquad K_n = iK', \qquad K_d = -K - iK'.$$

Now let the letters q, r, t, v be any permutation of the letters s, c, d, n. Then the elliptic function qr z is defined by the following statements:

(i) qr z is a doubly periodic function with a simple zero at K_q and a simple pole at K_r.

(ii) The step $K_r - K_q$ from the zero to the pole is a half period; those of the numbers K_c, K_n, K_d, which are different from $K_r - K_q$, are only quarter periods.

(iii) The coefficient of the leading term in the expansion of qr z near $z = 0$ is unity.

With regard to (iii), the leading term at the origin is z, $1/z$, 1, according as the origin is a zero, a pole, or an ordinary point.

The following table shows the poles and periods of all 12 functions:

Periods	Pole K_s	Pole K_c	Pole K_n	Pole K_d
$2K$, $4iK'$	cs z	sc z	dn z	nd z
$4K$, $2iK'$	ns z	dc z	sn z	cd z
$4K$, $2K + 2iK'$	ds z	nc z	cn z	sd z

It follows from Liouville's theorem that a doubly periodic function devoid of poles is simply a constant. Now the product (pq z)(qp z) is such a function, since each factor has a simple zero at the (simple) poles of the other. Thus the product (pq z)(qp z) is a constant, and, in view of (iii) above, this constant is unity. Therefore the functions pq z, qp z are reciprocals. If we agree that pp z is to be replaced always by unity, then a similar argument shows that

$$\text{pq } z = \text{pr } z/\text{qr } z$$

however, p, q, r may be chosen from s, c, d, n.

A more ambitious application of the same principle is to show that

$$\left(\frac{\text{sn } z \text{ dn } z}{\text{cn } z}\right)^2 = \frac{1 - \text{cn } 2z}{1 + \text{cn } 2z},$$

which can be done by showing that the quotient of the two sides is a function devoid of poles.

Of the above 12 functions, the 6 which have a pole or a zero at the origin are odd functions of z, the remaining 6 are even functions. Thus, for example,

$$\text{cs } z = - \text{cs } (-z), \qquad \text{cn } z = \text{cn } (-z).$$

To see this, observe that cs z and cs $(-z)$ have the same poles and zeros, and therefore cs $z \div$ cs $(-z)$ is a constant which must be -1 in virtue of property (iii) in the definition.

Just as a circular function such as sin z repeats its values in infinite strips of breadth 2π, so an elliptic function such as sn z repeats its values in a chequer pattern of rectangles whose sides are of lengths $4K$ and $2K'$. (A similar remark applies to all 12 functions). Thus sn z assumes the same value at the *congruent* points $z + p \,.\, 4K + n \,.\, 2iK'$. Within such a rectangle sn z has a simple pole at the point which is congruent with iK', residue $m^{-1/2}$, and a simple pole at the point congruent with $2K + iK'$, residue $-m^{-1/2}$.

The following list gives some critical values.

$$\text{cs } (iK') = -i, \qquad \text{ns } K = 1, \qquad \text{ns } (K + iK') = m^{1/2},$$

$$\text{ds } (iK') = -im^{1/2}, \quad \text{ds } K = m_1^{1/2}, \quad \text{cs } (K + iK') = -im_1^{1/2}.$$

The effect of a quarter period step in the lattice is clear from the diagram. Thus, for example,

$$\text{cn}\,(K + z) = \text{constant} \times \text{sd}\, z,$$

for the step K (or K_c) to the right changes c to s and n to d.

To discover the constant put $z = -K$, then

$$1 = \text{constant} \times \text{sd}\,(-K) = -\text{constant} \times \text{sd}\, K,$$

so that from the above table of critical values we get

$$\text{cn}\,(K + z) = -m_1^{1/2}\,\text{sd}\, z.$$

Similarly, we can show that

$$\text{cn}\,(iK' + z) = -im^{-1/2}\,\text{ds}\, z, \qquad \text{sn}\,(iK' + z) = m^{-1/2}\,\text{ns}\, z,$$

$$\text{sn}\,(K + z) = \text{cd}\, z.$$

Consideration of the lattice also leads to Jacobi's imaginary transformation (p. 21) as follows. The interchange of m and m_1 interchanges K and K'. This interchange can be effected by rotating the axes of reference through a right angle, and at the same time interchanging the labels c and n in the lattice, and so

$$\text{sn}\,(iz \mid m) = i\,\text{sc}\,(z \mid m_1), \qquad \text{cn}\,(iz \mid m) = \text{nc}\,(z \mid m_1),$$

$$\text{dn}\,(iz \mid m) = \text{dc}\,(z \mid m_1)$$

Simple reasoning on such lines will serve to determine many of the formulae in the pages which follow.

Numerical Examples

Let $f(a)$, $f(a + \omega)$ be successive tabulated values of a function. When third order differences are negligible, we have

(A) $$f(a + x\omega) = f(a) + x\{\Delta' - \tfrac{1}{2}(1 - x)\Delta''\},$$

where Δ' is the first, and Δ'' the second, difference taken with their proper signs. Alternatively

(B) $$f(a + \omega - x\omega) = f(a + \omega) - x\{\Delta' + \tfrac{1}{2}(1 - x)\Delta''\}.$$

In using the five-figure tables it is generally not important to distinguish between forward, backward, or mean values of the second difference.*

The existence of the two formulae (A) and (B) implies that we can always suppose that $x \leq 0\cdot 5$. In either formula the term $\frac{1}{2}(1 - x)\Delta''$ may be regarded as a correction to Δ'; it is quite simple to make a critical table showing this correction.†

Interpolation down the table (i.e. for u) is everywhere simple. Interpolation across the table (i.e. for m) is generally easier when u is less than $\frac{1}{2}K$. We can always replace u by $u - K$ or by $u - 2K$ by means of the formulae given on p. 15.

(1) Find sn $(0\cdot 54927 \mid 0\cdot 7)$.

The argument lies between $0\cdot 54$ and $0\cdot 55$, nearer the latter so we use (B) with $\Delta' = 784$, $\Delta'' = -7$ and $x = 0\cdot 073$. Treating the tabulated values as integers the required number is

$$50715 - 0\cdot 073\{784 + \tfrac{1}{2} \times 0\cdot 927 \times (-7)\}$$

$$= 50715 - 0\cdot 073 \times 780\cdot 8 = 50658.$$

Thus sn $(0\cdot 54927 \mid 0\cdot 7) = 0\cdot 50658$.

(2) Find sn $(4\cdot 7 \mid 0\cdot 7)$.

The argument lies outside those tabulated so we reduce it. From p. 108, $4\cdot 7 - 2K = 4\cdot 7 - 4\cdot 15073 = 0\cdot 54927$ and from example (1) sn $(4\cdot 7 \mid 0\cdot 7) = -0\cdot 50658$.

(3) Find cn $(0\cdot 54 \mid 0\cdot 9604)$.

$$\begin{array}{lll} \text{cn}\,(0\cdot 54 \mid 0\cdot 8) = 86763 & & \\ & +121 & \\ \text{cn}\,(0\cdot 54 \mid 0\cdot 9) = 86884 & & -1. \\ & +120 & \\ \text{cn}\,(0\cdot 54 \mid 1\cdot 0) = 87004 & & \end{array}$$

*For a full discussion of formulae and methods of interpolation, direct and inverse, see Milne-Thomson, *The calculus of finite differences*, London, (1933).

†Milne-Thomson, *Standard table of square roots*, London, (1928).

Thus $\Delta' = 120$, $\Delta'' = -1$ and therefore

$$\text{cn}\,(0\cdot54 \mid 0\cdot9604) = 0\cdot86956$$

(4) Find cn $(1\cdot15564 \mid 0\cdot99)$.

We first calculate $\text{cn}\,(1\cdot15564 \mid 0\cdot9) = 55704$

$$\text{cn}\,(1\cdot15564 \mid 1\cdot0) = 57292$$

so that $\Delta' = +1588$. Instead of forming cn $(1\cdot15564 \mid 0\cdot8)$ in order to calculate Δ'' we observe that in this and similar cases, it is sufficient to form the second difference of the nearest tabular values in this case cn $(1\cdot16 \mid 0\cdot8)$, cn $(1\cdot16 \mid 0\cdot9)$, cn $(1\cdot16 \mid 1\cdot0)$ giving $\Delta'' = -28$, and

$$\text{cn}\,(1\cdot15664 \mid 0\cdot99) = 57292 - 0\cdot1\{1588 + \tfrac{1}{2} \times 0\cdot9(-28)\}$$

$$= 0\cdot57134$$

By similar steps we find dn $(1\cdot15664 \mid 0\cdot99) = 0\cdot57721$.

(5) Find sn $(2\cdot54 \mid 0\cdot99)$.

We have

$$\text{sn}\,u = \frac{\text{cn}\,(K - u)}{\text{dn}\,(K - u)}\,.$$

From p. 106 $K(0\cdot99) = 3\cdot69564$, $K - u = 1\cdot15564$. Using (4)

$$\text{sn}\,(2\cdot54 \mid 0\cdot99) = \frac{0\cdot57134}{0\cdot57721} = 0\cdot98983.$$

(6) $\text{sn}\,[0\cdot75, (0\cdot4)^{1/2}] = \text{sn}\,(0\cdot75 \mid 0\cdot4) = 0\cdot66316$.

(7) Given $u = 0\cdot6$, $m = 0\cdot25$, find dn $(u \mid m_1)$.

Here $m = 0\cdot25$, $m_1 = 0\cdot75$, dn $(0\cdot6 \mid 0\cdot7) = 0\cdot88986$, $\Delta' = -1550$, $\Delta'' = 5$ and dn $(0\cdot6 \mid 0\cdot75) = 0\cdot88210$.

(8) Find cn $(2^{1/2} \mid 2)$.

Here the parameter exceeds unity. We therefore reduce to the reciprocal parameter, p. 19, which gives

$$\text{cn}\,(2^{1/2} \mid 2) = \text{dn}\,(2^{1/2} \times 2^{1/2} \mid \tfrac{1}{2}) = \text{dn}\,(2 \mid 0\cdot5) = 0\cdot71086.$$

(9) Find dn $(8^{-1/2} \mid -1)$.

Here the parameter is negative. With the notation of p. 19 we have

$$\mu = \tfrac{3}{2}, \qquad \mu_1 = \tfrac{1}{2}, \qquad v = 8^{-1/2} \times 2^{1/2} = \tfrac{1}{2} \qquad \text{and}$$

$$\mathrm{dn}\,(8^{-1/2} \mid -1) = \mathrm{nd}\,(0\cdot 5 \mid 0\cdot 5) = \frac{1}{\mathrm{dn}\,(0\cdot 5 \mid 0\cdot 5)} = 1\cdot 0605.$$

(10) Find $\wp(0\cdot 5;\ 16,\ 0)$, Weierstrass's $\wp$-function.

From p. 23 we see that e_1, e_2, e_3 are the solutions of the equation $4x^3 - 16x = 0$, so that

$$e_1 = 2, \qquad e_2 = 0, \qquad e_3 = -2, \qquad m = \frac{e_2 - e_3}{e_1 - e_3} = \frac{1}{2},$$

$$\wp(0\cdot 5;\ 16,\ 0) = -2 + 4\,\mathrm{ns}^2\,(1\cdot 0 \mid 0\cdot 5)$$

$$= -2 + \frac{4}{(0\cdot 803)^2} = 4\cdot 2034$$

(11) Find $\wp(0\cdot 2;\ -52,\ -136)$.

Here e_1, e_2, e_3 are the solutions of

$$4x^3 + 52x + 136 = 4(x + 2)(x^2 - 2x + 17) = 0,$$

$$e_2 = -2, \quad H^2 = 2e_2^2 + \frac{g_3}{4e_2} = 25, \quad m = \frac{1}{2} - \frac{3e_2}{4H} = 0\cdot 8$$

$$\wp(0\cdot 2;\ -52,\ -136) = -2 + 5 \cdot \frac{1 + \mathrm{cn}\,(0\cdot 894427 \mid 0\cdot 8)}{1 - \mathrm{cn}\,(0\cdot 894427 \mid 0\cdot 8)}$$

$$= -2 + 5 \cdot \frac{1\cdot 68641}{0\cdot 31359} = 24\cdot 889$$

The foregoing examples concern *direct* interpolation. We now consider *inverse* interpolation. Taking formula (A) the problem is essentially to find $a + x\omega$ given $f(a + x\omega)$. We shall use the method of successive approximation as follows. Let

$$f(a + x\omega) - f(a) = b.$$

The number b is known and cannot exceed $f(a+\omega) - f(a)$, i.e. Δ'. If we call x_1 the first and x_2 the second approximation to x, we have

$$\text{(C)} \qquad x_1 = \frac{b}{\Delta'}, \qquad x_2 = \frac{b}{\Delta' - \frac{1}{2}(1 - x_1)\Delta''}.$$

The process can be continued by writing x_2 for x_1 in the denominator of the second fraction to yield a third approximation x_3. In general a third application is unnecessary and indeed x_1 is often sufficiently accurate. Similarly given $f(a + \omega - x\omega)$, formula (B) leads to the successive approximations.

$$\text{(D)} \qquad x_1 = \frac{c}{\Delta'}, \qquad x_2 = \frac{c}{\Delta' + \frac{1}{2}(1 - x_1)\Delta''},$$

where $c = f(a+\omega) - f(a + \omega - x\omega)$.

(12) Find $u = \text{sn}^{-1}(0{\cdot}62 \mid 0{\cdot}6)$.

Here $\text{sn}\, u = 0{\cdot}62$. Searching the tables we find

$$\text{sn}\,(0{\cdot}69 \mid 0{\cdot}6) = 61344$$

$$\text{sn}\,(0{\cdot}70 \mid 0{\cdot}6) = 62035$$

with $\Delta' = 691$, $\Delta'' = -7$. The required value of u is therefore nearer $0{\cdot}70$. We therefore use formula (D) with $c = 35$. Then

$$x_1 = \frac{35}{691} = 0{\cdot}05065, \qquad x_2 = \frac{35}{687{\cdot}7} = 0{\cdot}05089$$

Since $\omega = 0{\cdot}01$, the corresponding approximations to u are

$$u_1 = 0{\cdot}70 - x_1 \times 0{\cdot}01 = 0{\cdot}69949\,(4)$$

$$u_2 = 0{\cdot}70 - x_2 \times 0{\cdot}01 = 0{\cdot}69949\,(1)$$

Thus to five figures

$$u = 0{\cdot}69949.$$

(13) Find $u = \text{sc}^{-1}(0{\cdot}55512 \mid 0{\cdot}6)$.

Here $\text{sc}\, u = 0{\cdot}55512$ but it is not necessary to perform calculations with $\text{sc}\, u$. From p. 17

$$\mathrm{cn}^2 u = \frac{1}{1 + \mathrm{sc}^2 u} = \frac{1}{1 + (0 \cdot 5512)^2} = 0 \cdot 76443,$$

$$\mathrm{cn}\,(u \mid 0 \cdot 6) = 0 \cdot 87432, \qquad u = 0 \cdot 52$$

(14) $$\int_0^{0 \cdot 62} \frac{dx}{[(1 - x^2)(1 - (3/5)x^2)]^{1/2}} = \mathrm{sn}^{-1}\,(0 \cdot 62 \mid 0 \cdot 6)$$

$$= 0 \cdot 69949 \text{ from } (12)$$

(15) $$\int_0^{\pi/6} \frac{d\theta}{(1 - \sin^2 (\pi/4) \sin^2 \theta)^{1/2}} = \mathrm{sn}^{-1}\,(0 \cdot 5 \mid 0 \cdot 5)$$

$$= 0 \cdot 53562$$

(16) Evaluate $I = \int_5^{47} \frac{dx}{[(x + 1)(x + 2)(x + 3)]^{1/2}}$

$$I = \int_5^{\infty} - \int_{47}^{\infty} \text{ of the same integrand.}$$

Comparing with the table of integrals with denominator $[(x - \alpha)(x - \beta)(x - \gamma)]^{1/2}$ we see that

$$\alpha = -1, \quad \beta = -2, \quad \gamma = -3, \quad \lambda = 2^{1/2}, \quad m = 0 \cdot 5$$

Therefore $I = 2^{1/2}\{\mathrm{sn}^{-1}\,(0 \cdot 5 \mid 0 \cdot 5) - \mathrm{sn}^{-1}\,(0 \cdot 2 \mid 0 \cdot 5)\}$

$$= 2^{1/2}\{0 \cdot 53562 - 0 \cdot 20204\} = 0 \cdot 47175.$$

(17) Evaluate $I = \int_0^{0 \cdot 62} \frac{x^2\, dx}{[(1 - x^2)(1 - (3/5)x^2)]^{1/2}}$

This is an elliptic integral of the second kind, and is reduced by the substitution appropriate to the pattern of the corresponding integral of the first kind, cf. (14), namely

$$x = \mathrm{sn}\,(u \mid 0 \cdot 6), \qquad 0 \cdot 62 = \mathrm{sn}\,(v \mid 0 \cdot 6),$$

which give $v = 0{\cdot}69949$, from (12), and

$$I = \int_0^v \operatorname{sn}^2 u\, du = \frac{5}{3}\int_0^v (1 - \operatorname{dn}^2 u)\, du = \frac{5}{3}\{v - E(v)\}$$

$$= \frac{5}{3}\left\{v - v\frac{E}{K} - Z(v \mid 0{\cdot}6)\right\}$$

in terms of Jacobi's zeta function. From* p. 108 we find E and K for the parameter $0{\cdot}6$ and $E/K = 0{\cdot}6660082$. From the table, $Z(0{\cdot}69949 \mid 0{\cdot}6) = 0{\cdot}1744699$ and so $I = 0{\cdot}09859$ to five places. This is the greatest number of places justified by the calculation, since v was obtained from five-figure tables.

(18) Given $\dfrac{K'}{5} = \dfrac{K}{4}$, find m, K, K'.

We have $q = \exp(-\pi K'/K) = \exp(-5\pi/4) = 0{\cdot}01970287$. From the table of q, by inverse interpolation using successive approximation, we get $m = 0{\cdot}27050969$, and now from the same table, by direct interpolation,

$$K = 1{\cdot}6970332, \qquad K' = 2{\cdot}1213116.$$

That K, K' and m can all be determined from the ratio of K to K' is an important fact which has many applications (see for example p. 33).

Complete Elliptic Integrals

$$k^2 = m, \qquad m_1 = 1 - m, \qquad k'^2 = m_1 = 1 - k^2$$

$$K = \int_0^1 \frac{dx}{\{(1 - x^2)(1 - m\,x^2)\}^{1/2}},$$

$$K' = \int_0^1 \frac{dx}{\{(1 - x^2)(1 - m_1\,x^2)\}^{1/2}}$$

*In this case the table on p. 33 could be used.

$$E = \int_0^1 \left\{\frac{1 - m x^2}{1 - x^2}\right\}^{1/2} dx, \qquad E' = \int_0^1 \left\{\frac{1 - m_1 x^2}{1 - x^2}\right\}^{1/2} dx$$

$$KE' + K'E - KK' = \tfrac{1}{2}\pi$$

$$q = \exp\left(-\pi \frac{K'}{K}\right), \qquad q_1 = \exp\left(-\pi \frac{K}{K'}\right)$$

$$\log_{10} \frac{1}{q} \log_{10} \frac{1}{q_1} = 1{\cdot}86152\ 28349$$

Series

$$\operatorname{sn} u = \frac{2\pi}{m^{1/2}K} \sum_{s=0}^{\infty} \frac{q^{s+1/2}}{1 - q^{2s+1}} \sin\left\{\frac{(2s+1)\pi u}{2K}\right\}$$

$$\operatorname{cn} u = \frac{2\pi}{m^{1/2}K} \sum_{s=0}^{\infty} \frac{q^{s+1/2}}{1 + q^{2s+1}} \cos\left\{\frac{(2s+1)\pi u}{2K}\right\}$$

$$\operatorname{dn} u = \frac{\pi}{2K} + \frac{2\pi}{K} \sum_{s=1}^{\infty} \frac{q^s}{1 + q^{2s}} \cos \frac{s\pi u}{K}$$

$$\operatorname{sn} u = u - (1 + m) \frac{u^3}{3!} + (1 + 14m + m^2) \frac{u^5}{5!}$$

$$- (1 + 135m + 135m^2 + m^3) \frac{u^7}{7!} + \cdots$$

$$\operatorname{cn} u = 1 - \frac{u^2}{2!} + (1 + 4m) \frac{u^4}{4!} - (1 + 44m + 16m^2) \frac{u^6}{6!} + \cdots$$

$$\operatorname{dn} u = 1 - m \frac{u^2}{2!} + m(m + 4) \frac{u^4}{4!}$$

$$- m(m^2 + 44m + 16) \frac{u^6}{6!} + \cdots$$

	sn u	cn u	dn u
Periods	$4K,\ 2iK'$	$4K,\ 2K + 2iK'$	$2K,\ 4iK'$
Zeros	$0,\ 2K$	$K,\ 3K$	$K + iK',\ K + 3iK'$
Poles	$iK',\ 2K + iK'$	$iK',\ 2K + iK'$	$iK',\ 3iK'$
Residues	$m^{-1/2},\ -m^{-1/2}$	$-im^{-1/2},\ im^{-1/2}$	$-i,\ i$

All points congruent to the above poles and zeros by the addition or subtraction of any integral multiples of the periods are likewise poles and zeros.

sn u is an odd function of u, i.e. sn $(-u) = -$sn u.
cn u, dn u are even functions of u, i.e. cn $(-u) =$ cn u,
dn $(-u) =$ dn u

Argument	sn	cn	dn
u	$\operatorname{sn} u$	$\operatorname{cn} u$	$\operatorname{dn} u$
$-u$	$-\operatorname{sn} u$	$\operatorname{cn} u$	$\operatorname{dn} u$
$u + K$	$\operatorname{cd} u$	$-m_1^{1/2} \operatorname{sd} u$	$m_1^{1/2} \operatorname{nd} u$
$u - K$	$-\operatorname{cd} u$	$m_1^{1/2} \operatorname{sd} u$	$m_1^{1/2} \operatorname{nd} u$
$K - u$	$\operatorname{cd} u$	$m_1^{1/2} \operatorname{sd} u$	$m_1^{1/2} \operatorname{nd} u$
$u + 2K$	$-\operatorname{sn} u$	$-\operatorname{cn} u$	$\operatorname{dn} u$
$u - 2K$	$-\operatorname{sn} u$	$-\operatorname{cn} u$	$\operatorname{dn} u$
$2K - u$	$\operatorname{sn} u$	$-\operatorname{cn} u$	$\operatorname{dn} u$
$u + iK'$	$m^{-1/2} \operatorname{ns} u$	$-im^{-1/2} \operatorname{ds} u$	$-i \operatorname{cs} u$
$u + 2iK'$	$\operatorname{sn} u$	$-\operatorname{cn} u$	$-\operatorname{dn} u$
$u + K + iK'$	$m^{-1/2} \operatorname{dc} u$	$-im_1^{1/2} m^{-1/2} \operatorname{nc} u$	$im_1^{1/2} \operatorname{sc} u$
$u + 2K + 2iK'$	$-\operatorname{sn} u$	$\operatorname{cn} u$	$-\operatorname{dn} u$

The functions cd u, sd u, nd u may often be conveniently calculated from the formulae

$$\operatorname{cd} u = \operatorname{sn}(K - u), \qquad \operatorname{sd} u = m_1^{-1/2} \operatorname{cn}(K - u),$$

$$\operatorname{nd} u = m_1^{-1/2} \operatorname{dn}(K - u)$$

Special Values of the Argument

u	$\mathrm{sn}\, u$	$\mathrm{cn}\, u$	$\mathrm{dn}\, u$
0	0	1	1
$\frac{1}{2}K$	$\frac{1}{(1+m_1^{1/2})^{1/2}}$	$\frac{m_1^{1/4}}{(1+m_1^{1/2})^{1/2}}$	$m_1^{1/4}$
K	1	0	$m_1^{1/2}$
$2K$	0	-1	1
$\frac{1}{2}iK'$	$im^{-1/4}$	$\frac{(1+m^{1/2})^{1/2}}{m^{1/4}}$	$(1+m^{1/2})^{1/2}$
iK'	∞	∞	∞
$2iK'$	0	-1	-1
$K+iK'$	$m^{-1/2}$	$-i(m_1/m)^{1/2}$	0
$2K+2iK'$	0	1	-1

Relations between the Squares of the Jacobian Functions

$$\mathrm{sn}^2 u + \mathrm{cn}^2 u = 1, \qquad \mathrm{dn}^2 u + m\ \mathrm{sn}^2 u = 1,$$

$$\mathrm{dn}^2 u - m\ \mathrm{cn}^2 u = m_1$$

$$\mathrm{ns}^2 u - \mathrm{cs}^2 u = 1, \qquad \mathrm{ds}^2 u - \mathrm{cs}^2 u = m_1,$$

$$\mathrm{ns}^2 u - \mathrm{ds}^2 u = m$$

$$m_1 \operatorname{sd}^2 u + \operatorname{cd}^2 u = 1, \qquad \operatorname{nd}^2 u - m \operatorname{sd}^2 u = 1,$$

$$m \operatorname{cd}^2 u + m_1 \operatorname{nd}^2 u = 1$$

$$\operatorname{nc}^2 u - \operatorname{sc}^2 u = 1, \qquad \operatorname{dc}^2 u - m_1 \operatorname{sc}^2 u = 1,$$

$$\operatorname{dc}^2 u - m_1 \operatorname{nc}^2 u = m$$

With the aid of these identities the square of any function can be expressed in terms of the square of any other. In particular

$$\operatorname{sn}^2 u = \frac{1}{1 + \operatorname{cs}^2 u} = \frac{1}{m + \operatorname{ds}^2 u},$$

$$\operatorname{cn}^2 u = \frac{1}{1 + \operatorname{sc}^2 u} = \frac{m_1}{\operatorname{dc}^2 u - m},$$

$$\operatorname{dn}^2 u = \frac{1}{1 + m \operatorname{sd}^2 u} = \frac{m_1}{1 - m \operatorname{cd}^2 u}$$

It follows that the tables suffice to find the value of u corresponding to a given value of cs u, sc u, ds u, sd u, cd u, dc u.

Double and Half Arguments

$$\operatorname{sn} 2u = \frac{2\operatorname{sn} u \cdot \operatorname{cn} u \cdot \operatorname{dn} u}{1 - m \operatorname{sn}^4 u} = \frac{2\operatorname{sn} u \cdot \operatorname{cn} u \cdot \operatorname{dn} u}{\operatorname{cn}^2 u + \operatorname{sn}^2 u \cdot \operatorname{dn}^2 u}$$

$$\operatorname{cn} 2u = \frac{\operatorname{cn}^2 u - \operatorname{sn}^2 u \cdot \operatorname{dn}^2 u}{1 - m \operatorname{sn}^4 u} = \frac{\operatorname{cn}^2 u - \operatorname{sn}^2 u \cdot \operatorname{dn}^2 u}{\operatorname{cn}^2 u + \operatorname{sn}^2 u \cdot \operatorname{dn}^2 u}$$

$$\operatorname{dn} 2u = \frac{\operatorname{dn}^2 u - m \operatorname{sn}^2 u \cdot \operatorname{cn}^2 u}{1 - m \operatorname{sn}^4 u} = \frac{\operatorname{dn}^2 u + \operatorname{cn}^2 u(\operatorname{dn}^2 u - 1)}{\operatorname{dn}^2 u - \operatorname{cn}^2 u(\operatorname{dn}^2 u - 1)}$$

$$\frac{1 - \operatorname{cn} 2u}{1 + \operatorname{cn} 2u} = \frac{\operatorname{sn}^2 u \operatorname{dn}^2 u}{\operatorname{cn}^2 u}, \qquad \frac{1 - \operatorname{dn} 2u}{1 + \operatorname{dn} 2u} = \frac{m \operatorname{sn}^2 u \cdot \operatorname{cn}^2 u}{\operatorname{dn}^2 u}$$

$$\mathrm{sn}^2 \frac{1}{2} u = \frac{1 - \mathrm{cn}\, u}{1 + \mathrm{dn}\, u}, \qquad \mathrm{cn}^2 \frac{1}{2} u = \frac{\mathrm{dn}\, u + \mathrm{cn}\, u}{1 + \mathrm{dn}\, u}$$

$$\mathrm{dn}^2 \frac{1}{2} u = \frac{m_1 + \mathrm{dn}\, u + m\, \mathrm{cn}\, u}{1 + \mathrm{dn}\, u}$$

Addition Theorems

$$\mathrm{sn}\,(u + v) = \frac{\mathrm{sn}\, u \cdot \mathrm{cn}\, v \cdot \mathrm{dn}\, v + \mathrm{sn}\, v \cdot \mathrm{cn}\, u \cdot \mathrm{dn}\, u}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{cn}\,(u + v) = \frac{\mathrm{cn}\, u \cdot \mathrm{cn}\, v - \mathrm{sn}\, u \cdot \mathrm{dn}\, u \cdot \mathrm{sn}\, v \cdot \mathrm{dn}\, v}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{dn}\,(u + v) = \frac{\mathrm{dn}\, u \cdot \mathrm{dn}\, v - m\, \mathrm{sn}\, u \cdot \mathrm{cn}\, u \cdot \mathrm{sn}\, v \cdot \mathrm{cn}\, v}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{sn}\,(u + v) \cdot \mathrm{sn}\,(u - v) = \frac{\mathrm{sn}^2 u - \mathrm{sn}^2 v}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{sn}\,(u + v) \cdot \mathrm{cn}\,(u - v) = \frac{\mathrm{sn}\, u \cdot \mathrm{cn}\, u \cdot \mathrm{dn}\, v + \mathrm{sn}\, v \cdot \mathrm{cn}\, v \cdot \mathrm{dn}\, u}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{sn}\,(u + v) \cdot \mathrm{dn}\,(u - v) = \frac{\mathrm{sn}\, u \cdot \mathrm{dn}\, u \cdot \mathrm{cn}\, v + \mathrm{sn}\, v \cdot \mathrm{dn}\, v \cdot \mathrm{cn}\, u}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{cn}\,(u + v) \cdot \mathrm{cn}\,(u - v) = \frac{\mathrm{cn}^2 u - \mathrm{sn}^2 v \cdot \mathrm{dn}^2 u}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{cn}\,(u + v) \cdot \mathrm{dn}\,(u - v) = \frac{\mathrm{cn}\, u \cdot \mathrm{dn}\, u \cdot \mathrm{cn}\, v \cdot \mathrm{dn}\, v - m_1\, \mathrm{sn}\, u \cdot \mathrm{sn}\, v}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

$$\mathrm{dn}\,(u + v) \cdot \mathrm{dn}\,(u - v) = \frac{\mathrm{dn}^2 u - m\, \mathrm{cn}^2 u \cdot \mathrm{sn}^2 v}{1 - m\, \mathrm{sn}^2 u \cdot \mathrm{sn}^2 v}$$

Negative parameter

Let m be a positive number and write

$$\mu = \frac{m}{1 + m}, \qquad \mu_1 = \frac{1}{1 + m}, \qquad v = \frac{u}{\mu_1^{1/2}}$$

Then

$$\operatorname{sn}(u \mid -m) = \mu_1^{1/2} \operatorname{sd}(v \mid \mu), \qquad \operatorname{cn}(u \mid -m) = \operatorname{cd}(v \mid \mu),$$

$$\operatorname{dn}(u \mid -m) = \operatorname{nd}(v \mid \mu)$$

Thus elliptic functions with negative parameter can be made to depend upon elliptic functions with a positive parameter. Note that $0 < \mu < 1$.

Reciprocal parameter

$$\operatorname{sn}(u \mid m) = m^{-1/2} \operatorname{sn}(u\, m^{1/2} \mid m^{-1}),$$

$$\operatorname{cn}(u \mid m) = \operatorname{dn}(u\, m^{1/2} \mid m^{-1}),$$

$$\operatorname{dn}(u \mid m) = \operatorname{cn}(u\, m^{1/2} \mid m^{-1})$$

This is Jacobi's *real transformation*. If $m > 1$, then $m^{-1} < 1$, and therefore elliptic functions whose parameter is greater than 1 can be made to depend upon those whose parameter is less than 1. It follows that there is no loss of generality in supposing $0 \le m \le 1$.

Decrease of parameter

$$\mu = \left(\frac{1 - m_1^{1/2}}{1 + m_1^{1/2}}\right)^2, \qquad v = \frac{u}{1 + \mu^{1/2}}$$

$$\operatorname{sn}(u \mid m) = \frac{(1 + \mu^{1/2}) \operatorname{sn}(v \mid \mu)}{1 + \mu^{1/2} \operatorname{sn}^2(v \mid \mu)},$$

$$\operatorname{cn}(u \mid m) = \frac{\operatorname{cn}(v \mid \mu) \cdot \operatorname{dn}(v \mid \mu)}{1 + \mu^{1/2} \operatorname{sn}^2(v \mid \mu)}$$

$$\operatorname{dn}(u \mid m) = \frac{1 - \mu^{1/2} \operatorname{sn}^2(v \mid \mu)}{1 + \mu^{1/2} \operatorname{sn}^2(v \mid \mu)}$$

$$= \frac{\operatorname{dn}(2v \mid \mu) + \mu^{1/2} \operatorname{cn}(2v \mid \mu)}{1 + \mu^{1/2}}$$

This is Gauss's transformation or the *descending* Landen transformation, whereby elliptic functions are made to depend upon elliptic functions with a smaller parameter. The rate of decrease is rapid. Thus if $m = 0{\cdot}64$, we find $\mu = 1/16$.

Increase of parameter

$$\mu_1 = \left(\frac{1 - m^{1/2}}{1 + m^{1/2}}\right)^2, \qquad \mu = \frac{4m^{1/2}}{(1 + m^{1/2})^2}, \qquad v = \frac{u}{1 + \mu_1^{1/2}}$$

$$\operatorname{sn}(u \mid m) = (1 + \mu_1^{1/2}) \frac{\operatorname{sn}(v \mid \mu) \cdot \operatorname{cn}(v \mid \mu)}{\operatorname{dn}(v \mid \mu)},$$

$$\operatorname{cn}(u \mid m) = \frac{1 - (1 + \mu_1^{1/2}) \operatorname{sn}^2(v \mid \mu)}{\operatorname{dn}(v \mid \mu)},$$

$$\operatorname{dn}(u \mid m) = \frac{1 - (1 - \mu_1^{1/2}) \operatorname{sn}^2(v \mid \mu)}{\operatorname{dn}(v \mid \mu)}$$

This is the *ascending* Landen transformation, whereby dependence on a larger parameter is secured. The rate of increase is rapid. Thus if $m = 0{\cdot}64$, we find $\mu = 80/81$.

Approximations

When the parameter m is so small that its square may be neglected, the following approximations may be used to calculate the elliptic functions in terms of circular functions.

$$\operatorname{sn}(u \mid m) = \sin u - \tfrac{1}{4} m \cos u\,(u - \sin u \cos u),$$

$$\operatorname{cn}(u \mid m) = \cos u + \tfrac{1}{4} m \sin u\,(u - \sin u \cos u),$$

$$\operatorname{dn}(u \mid m) = 1 - \tfrac{1}{2} m \sin^2 u$$

When the parameter m is so near unity that the square of the complementary parameter m_1 may be neglected, the following approximations may be used to calculate the elliptic functions in terms of hyperbolic functions.

$$\text{sn}\,(u \mid m) = \tanh u + \tfrac{1}{4}m_1 \,\text{sech}^2 u\,(\sinh u \cosh u - u)$$

$$\text{cn}\,(u \mid m) = \text{sech}\, u - \tfrac{1}{4}m_1 \tanh u \,\text{sech}\, u\,(\sinh u \cosh u - u)$$

$$\text{dn}\,(u \mid m) = \text{sech}\, u + \tfrac{1}{4}m_1 \tanh u \,\text{sech}\, u\,(\sinh u \cosh u + u)$$

The above results combined with one or more applications of Landen's transformation afford a means of direct calculation to any required degree of accuracy.

Complex Arguments

Jacobi's *imaginary transformation* (see p. 6) is

$$\text{sn}\,(iy \mid m) = i\,\text{sc}\,(y \mid m_1), \qquad \text{cn}\,(iy \mid m) = \text{nc}\,(y \mid m_1),$$

$$\text{dn}\,(iy \mid m) = \text{dc}\,(y \mid m_1)$$

If $z = x + iy$, the addition theorems then give with

$$s_1 = \text{sn}\,(x \mid m), \quad s_2 = \text{sn}\,(y \mid m_1),$$

$$c_1 = \text{cn}\,(x \mid m), \quad c_2 = \text{cn}\,(y \mid m_1),$$

$$d_1 = \text{dn}\,(x \mid m), \quad d_2 = \text{dn}\,(y \mid m_1)$$

$$\text{sn}\,(z \mid m) = \frac{s_1 \cdot d_2 + ic_1 \cdot d_1 \cdot s_2 \cdot c_2}{c_2^2 + ms_1^2 \cdot s_2^2}$$

$$\text{cn}\,(z \mid m) = \frac{c_1 \cdot c_2 - is_1 \cdot d_1 \cdot s_2 \cdot d_2}{c_2^2 + ms_1^2 \cdot s_2^2}$$

$$\text{dn}\,(z \mid m) = \frac{d_1 \cdot c_2 \cdot d_2 - ims_1 \cdot c_1 \cdot s_2}{c_2^2 + ms_1^2 \cdot s_2^2}$$

$$\frac{d}{du}\operatorname{sn} u = \operatorname{cn} u \cdot \operatorname{dn} u, \quad \frac{d}{du}\operatorname{cn} u = -\operatorname{sn} u \cdot \operatorname{dn} u,$$

$$\frac{d}{du}\operatorname{dn} u = -m \operatorname{sn} u \cdot \operatorname{cn} u$$

$$\frac{d}{du}\operatorname{cs} u = -\operatorname{ds} u \cdot \operatorname{ns} u, \quad \frac{d}{du}\operatorname{ns} u = -\operatorname{cs} u \cdot \operatorname{ds} u,$$

$$\frac{d}{du}\operatorname{ds} u = -\operatorname{cs} u \cdot \operatorname{ns} u$$

$$\frac{d}{du}\operatorname{sc} u = \operatorname{dc} u \cdot \operatorname{nc} u, \quad \frac{d}{du}\operatorname{nc} u = \operatorname{sc} u \cdot \operatorname{dc} u,$$

$$\frac{d}{du}\operatorname{dc} u = m_1 \operatorname{sc} u \cdot \operatorname{nc} u$$

$$\frac{d}{du}\operatorname{sd} u = \operatorname{cd} u \cdot \operatorname{nd} u, \quad \frac{d}{du}\operatorname{cd} u = -m_1 \operatorname{sd} u \cdot \operatorname{nd} u,$$

$$\frac{d}{du}\operatorname{nd} u = m \operatorname{sd} u \cdot \operatorname{cd} u$$

To solve Euler's equations of motion of a rigid body under no forces;

$$A\dot{p} - (B - C)qr = 0, \qquad B\dot{q} - (C - A)rp = 0,$$

$$C\dot{r} - (A - B)pq = 0,$$

where p, q, r are angular velocities and $A > B > C$, put

$$p = p_0 \operatorname{cn} [n(t - t_0) \mid m], \qquad q = h \operatorname{sn} [n(t - t_0) \mid m],$$

$$r = r_0 \operatorname{dn} [n(t - t_0) \mid m].$$

Substitution of these values in the differential equations gives

$$\frac{h^2}{p_0^2} = \frac{A(A - C)}{B(B - C)}, \qquad m = \frac{A - B}{B - C} \cdot \frac{Ap_0^2}{Cr_0^2},$$

$$n^2 = \frac{(A - C)(B - C)}{AB} r_0^2 .$$

Weierstrass's $\wp$-function

$$4x^3 - g_2x - g_3 = 4(x - e_1)(x - e_2)(x - e_3), \quad \Delta = g_2^3 - 27g_3^2$$

$$u = \wp^{-1}x = \int_x^\infty \frac{dx}{(4x^3 - g_2x - g_3)^{1/2}}, \qquad x = \wp(u;\, g_2,\, g_3)$$

$$\wp(u;\, g_2,\, g_3) = \lambda\wp(u\lambda^{1/2};\, g_2\lambda^{-2},\, g_3\lambda^{-3}),$$

$$\wp'^2u = 4\wp^3u - g_2\,\wp u - g_3$$

Half periods ω_1, ω_2, ω_3, where $\wp\omega_1 = e_1$, $\wp\omega_2 = e_2$, $\wp\omega_3 = e_3$ and $\omega_1 + \omega_2 + \omega_3 = 0$

$$\wp(u + v) + \wp u + \wp v = \frac{1}{4}\left[\frac{\wp'u - \wp'v}{\wp u - \wp v}\right]^2$$

Positive discriminant $\Delta > 0$

e_1, e_2, e_3 are real and we take $e_1 > e_2 > e_3$

$$\wp u = e_3 + (e_1 - e_3)\,\mathrm{ns}^2\left\{u(e_1 - e_3)^{1/2} \,\middle|\, \frac{e_2 - e_3}{e_1 - e_3}\right\}, \quad m = \frac{e_2 - e_3}{e_1 - e_3}$$

$$\wp'u = -2(e_1 - e_3)^{3/2}\,\mathrm{cn}\,\{u(e_1 - e_3)^{1/2}\}$$

$$\cdot\,\mathrm{dn}\,\{u(e_1 - e_3)^{1/2}\}\,\mathrm{ns}^3\,\{u(e_1 - e_3)^{1/2}\}$$

$$\omega_1 = \frac{K}{(e_1 - e_3)^{1/2}}, \qquad \omega_3 = \frac{iK'}{(e_1 - e_3)^{1/2}},$$

$$\eta_1 = (e_1 - e_3)^{1/2}\left\{E - \frac{e_1}{e_1 - e_3}K\right\}$$

$$\eta_3 = -i(e_1 - e_3)^{1/2}\left\{E' + \frac{e_3}{e_1 - e_3}K'\right\}$$

Negative discriminant $\Delta < 0$

Two of e_1, e_2, e_3 are complex. Take e_2 to be real. Write

$$H^2 = (e_2 - e_1)(e_2 - e_3) = 2e_2^2 + \frac{g_3}{4e_2}, \qquad m = \frac{1}{2} - \frac{3e_2}{4H}$$

$$\wp u = e_2 + H \frac{1 + \mathrm{cn}\,(2uH^{1/2})}{1 - \mathrm{cn}\,(2uH^{1/2})}$$

$$\wp' u = - \frac{4H^{3/2}\,\mathrm{sn}\,(2uH^{1/2})\,\mathrm{dn}\,(2uH^{1/2})}{\{1 - \mathrm{cn}\,(2uH^{1/2})\}^2}$$

Real half period $\omega_2 = \dfrac{K}{H^{1/2}}$, imaginary half period $\omega_2' = \dfrac{iK'}{H^{1/2}}$

Zero discriminant $\Delta = 0$

Two cases arise, either (A) $e_2 = e_3 = -\frac{1}{2}e_1$ so that

$$g_2 = 3e_1^2,\ g_3 = e_1^3,\ \omega_1 = \frac{\pi}{(6e_1)^{1/2}},\ K = \frac{1}{2}\pi,\ K' = \infty,\ \omega_3 = \infty$$

$$\wp u = -\frac{\pi^2}{12\omega_1^2} + \left(\frac{\dfrac{\pi}{2\omega_1}}{\sin \dfrac{\pi u}{2\omega_1}}\right)^2$$

or (*B*) $e_1 = e_2 = -\frac{1}{2}e_3$, $g_2 = 3e_3^2$, $g_3 = e_3^3$

$$\omega_3 = \frac{i\pi}{(12e_1)^{1/2}}, \qquad K = \infty, \qquad K' = \frac{1}{2}\pi, \qquad \omega_1 = \infty$$

$$\wp u = -2e_1 + \frac{3e_1}{\tanh^2\{u(3e_1)^{1/2}\}}$$

Integration of Jacobian Elliptic Functions

The following list wherein $\ln x$ denotes the natural logarithm of x and $\sin^{-1} x$, $\cos^{-1} x$ denote inverse trigonometric functions, gives the indefinite integrals of the 12 Jacobian elliptic functions.

$$\int \operatorname{sn} u\, du = m^{-1/2} \ln (\operatorname{dn} u - m^{1/2} \operatorname{cn} u),$$

$$\int \operatorname{cn} u\, du = m^{-1/2} \cos^{-1} (\operatorname{dn} u),$$

$$\int \operatorname{dn} u\, du = \sin^{-1} (\operatorname{sn} u),$$

$$\int \operatorname{ns} u\, du = \ln (\operatorname{ds} u - \operatorname{cs} u),$$

$$\int \operatorname{ds} u\, du = \ln (\operatorname{ns} u - \operatorname{cs} u),$$

$$\int \operatorname{cs} u\, du = \ln (\operatorname{ns} u - \operatorname{ds} u),$$

$$\int \operatorname{dc} u\, du = \ln (\operatorname{nc} u + \operatorname{sc} u),$$

$$\int \operatorname{nc} u\, du = m_1^{-1/2} \ln (\operatorname{dc} u + m_1^{1/2} \operatorname{sc} u),$$

$$\int \operatorname{sc} u\, du = m_1^{-1/2} \ln (\operatorname{dc} u + m_1^{1/2} \operatorname{nc} u),$$

$$\int \operatorname{cd} u\, du = m^{-1/2} \ln (\operatorname{nd} u + m^{1/2} \operatorname{sd} u),$$

$$\int \operatorname{sd} u\, du = (m m_1)^{-1/2} \sin^{-1} (-m^{1/2} \operatorname{cd} u),$$

$$\int \operatorname{nd} u\, du = m_1^{-1/2} \cos^{-1} (\operatorname{cd} u)$$

The integration of any rational function of the Jacobian functions can be made, by substitution and reduction, to depend upon the above 12 integrals together with the functions

$$E(u) = \int_0^u \mathrm{dn}^2\, t\, dt, \qquad \Pi(u, a) = \int_0^u \frac{m \,\mathrm{sn}\, a \,\mathrm{cn}\, a \,\mathrm{dn}\, a \,\mathrm{sn}^2\, t}{1 - m \,\mathrm{sn}^2\, a \,\mathrm{sn}^2\, t}\, dt,$$

which are known respectively as elliptic integrals of the second and third kinds.

Elliptic Integrals of the First Kind

An elliptic integral of the first kind is one of the form $\int dt/(R)^{1/2}$, where R is a cubic or quartic polynomial in t. In the following list if a, b, $(a^2 + b^2)^{1/2}$, x are all positive, the value of the inverse elliptic function is that which lies between 0 and K.

The value of each integral also indicates the substitution necessary to obtain it. Thus the first integral is obtained by the substitution

$$v = \frac{1}{a}\,\mathrm{sn}^{-1}\left(\frac{t}{b} \,\middle|\, \frac{b^2}{a^2}\right) \qquad \text{or} \qquad t = b\,\mathrm{sn}\left(av \,\middle|\, \frac{b^2}{a^2}\right)$$

which gives

$$dt = ab \,\mathrm{cn}\,(av)\,\mathrm{dn}\,(av)dv,$$
$$\{(a^2 - t^2)(b^2 - t^2)\}^{1/2} = a\,\mathrm{dn}\,(av)\, b\,\mathrm{cn}\,(av),$$

and the integral reduces to

$$\int_0^u dv = u, \qquad \text{where } x = b\,\mathrm{sn}\left(au \,\middle|\, \frac{b^2}{a^2}\right)$$

$$\int_0^x \frac{dt}{[(a^2 - t^2)(b^2 - t^2)]^{1/2}} = \frac{1}{a}\,\mathrm{sn}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{b^2}{a^2}\right), \qquad x < b < a$$

$$\int_x^\infty \frac{dt}{[(t^2 - a^2)(t^2 - b^2)]^{1/2}} = \frac{1}{a}\,\mathrm{ns}^{-1}\left(\frac{x}{a} \,\middle|\, \frac{b^2}{a^2}\right), \qquad b < a < x$$

$$\int_x^b \frac{dt}{[(a^2 - t^2)(b^2 - t^2)]^{1/2}} = \frac{1}{a}\,\mathrm{cd}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{b^2}{a^2}\right), \qquad x < b < a$$

$$\int_a^x \frac{dt}{[(t^2 - a^2)(t^2 - b^2)]^{1/2}} = \frac{1}{a} \operatorname{dc}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{b^2}{a^2}\right), \qquad b < a < x$$

$$\int_x^b \frac{dt}{[(t^2 + a^2)(b^2 - t^2)]^{1/2}}$$

$$= \frac{1}{(a^2 + b^2)^{1/2}} \operatorname{cn}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{b^2}{a^2 + b^2}\right), \qquad 0 < x < b$$

$$\int_b^x \frac{dt}{[(t^2 + a^2)(t^2 - b^2)]^{1/2}}$$

$$= \frac{1}{(a^2 + b^2)^{1/2}} \operatorname{nc}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{a^2}{a^2 + b^2}\right), \qquad 0 < b < x$$

$$\int_0^x \frac{dt}{[(t^2 + a^2)(b^2 - t^2)]^{1/2}}$$

$$= \frac{1}{(a^2 + b^2)^{1/2}} \operatorname{sd}^{-1}\left\{\frac{x(a^2 + b^2)^{1/2}}{ab} \,\middle|\, \frac{b^2}{a^2 + b^2}\right\}, \quad 0 < x < b$$

$$\int_x^\infty \frac{dt}{[(t^2 + a^2)(t^2 - b^2)]^{1/2}}$$

$$= \frac{1}{(a^2 + b^2)^{1/2}} \operatorname{ds}^{-1}\left\{\frac{x}{(a^2 + b^2)^{1/2}} \,\middle|\, \frac{a^2}{a^2 + b^2}\right\}, \quad 0 < b < x$$

$$\int_b^x \frac{dt}{[(a^2 - t^2)(t^2 - b^2)]^{1/2}} = \frac{1}{a} \operatorname{nd}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{a^2 - b^2}{a^2}\right), \quad b < x < a$$

$$\int_x^a \frac{dt}{[(a^2 - t^2)(t^2 - b^2)]^{1/2}} = \frac{1}{a} \operatorname{dn}^{-1}\left(\frac{x}{a} \,\middle|\, \frac{a^2 - b^2}{a^2}\right), \quad b < x < a$$

$$\int_0^x \frac{dt}{[(t^2 + a^2)(t^2 + b^2)]^{1/2}} = \frac{1}{a} \operatorname{sc}^{-1}\left(\frac{x}{b} \,\middle|\, \frac{a^2 - b^2}{a^2}\right), \quad b < a$$

$$\int_x^\infty \frac{dt}{[(t^2 + a^2)(t^2 + b^2)]^{1/2}} = \frac{1}{a} \operatorname{cs}^{-1}\left(\frac{x}{a} \,\middle|\, \frac{a^2 - b^2}{a^2}\right), \quad b < a$$

To evaluate $\int dx/(R)^{1/2}$ where

$$R = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$$

we resolve R into two real quadratic factors Q_1, Q_2. This can be done by inspection or, if necessary, by solving the biquadratic equation $R = 0$ by Ferrari's or other methods, (see p. 37). We can then find two numbers λ_1 and λ_2 such that $Q_1 + \lambda Q_2$ is a perfect square say

$$Q_1 + \lambda_1 Q_2 = A(x - \alpha)^2, \quad Q_1 + \lambda_2 Q_2 = B(x - \beta)^2, \quad \alpha > \beta$$

It then follows by solving for Q_1 and Q_2 that

$$Q_1 = A_1(x - \alpha)^2 + B_1(x - \beta)^2,$$

$$Q_2 = A_2(x - \alpha)^2 + B_2(x - \beta)^2.$$

The substitution $t = (x - \alpha)/(x - \beta)$ then gives

$$\frac{dx}{(R)^{1/2}} = \frac{1}{\alpha - \beta} \frac{dt}{\{(A_1t^2 + B_1)(A_2t^2 + B_2)\}^{1/2}}$$

and the integral is reduced to depend on one of the canonical forms given above.

The same method can be applied if $a_0 = 0$, provided that Q_2 is replaced by a linear factor.

Example $R = 3x^4 - 16x^3 + 24x^2 - 16x + 4$. We find $R = (3x^2 - 4x + 2)(x^2 - 4x + 2)$ and therefore

$$Q_1 + \lambda Q_2 = (3 + \lambda)x^2 - 4(1 + \lambda)x + 2(1 + \lambda)$$

This is a perfect square if $16(1 + \lambda)^2 = 8(3 + \lambda)(1 + \lambda)$, whence $\lambda = 1$ or -1 and $Q_1 + Q_2 = 4(x - 1)^2$, $Q_1 - Q_2 = 2x^2$. Thus

$$Q_1 = 2(x - 1)^2 + x^2, \qquad Q_2 = 2(x - 1)^2 - x^2$$

Put $t = (x - 1)/x$. Then

$$\int \frac{dx}{(R)^{1/2}} = \int \frac{dt}{[(2t^2 - 1)(2t^2 + 1)]^{1/2}}$$

$$= \frac{1}{2} \int \frac{dt}{[(t^2 - \frac{1}{2})(t^2 + \frac{1}{2})]^{1/2}}$$

When the denominator involves the square root of a cubic polynomial let

$$X = (x - \alpha)(x - \beta)(x - \gamma), \qquad \alpha > \beta > \gamma,$$

$$\lambda = \frac{2}{(\alpha - \gamma)^{1/2}}, \qquad m = \frac{\beta - \gamma}{\alpha - \gamma}, \qquad m_1 = \frac{\alpha - \beta}{\alpha - \gamma}$$

$$\int_x^\infty \frac{dx}{X^{1/2}} = \lambda \ \mathrm{sn}^{-1}\left\{\left(\frac{\alpha - \gamma}{x - \gamma}\right)^{1/2} \middle| \ m\right\}$$

$$\int_{-\infty}^x \frac{dx}{(-X)^{1/2}} = \lambda \ \mathrm{sn}^{-1}\left\{\left(\frac{\alpha - \gamma}{\alpha - x}\right)^{1/2} \middle| \ m_1\right\}$$

$$\int_\alpha^x \frac{dx}{X^{1/2}} = \lambda \ \mathrm{cn}^{-1}\left\{\left(\frac{\alpha - \beta}{x - \beta}\right)^{1/2} \middle| \ m\right\}$$

$$\int_x^\alpha \frac{dx}{(-X)^{1/2}} = \lambda \ \mathrm{cn}^{-1}\left\{\left(\frac{x - \beta}{\alpha - \beta}\right)^{1/2} \middle| \ m_1\right\}$$

$$\int_x^\beta \frac{dx}{X^{1/2}} = \lambda \ \mathrm{dn}^{-1}\left\{\left(\frac{\alpha - \beta}{\alpha - x}\right)^{1/2} \middle| \ m\right\}$$

$$\int_\beta^x \frac{dx}{(-X)^{1/2}} = \lambda \ \mathrm{dn}^{-1}\left\{\left(\frac{\beta - \gamma}{x - \gamma}\right)^{1/2} \middle| \ m_1\right\}$$

$$\int_\gamma^x \frac{dx}{X^{1/2}} = \lambda \ \mathrm{sn}^{-1}\left\{\left(\frac{x - \gamma}{\beta - \gamma}\right)^{1/2} \middle| \ m\right\}$$

$$\int_x^\gamma \frac{dx}{(-X)^{1/2}} = \lambda \ \mathrm{cn}^{-1}\left\{\left(\frac{\beta - \gamma}{\beta - x}\right)^{1/2} \middle| \ m_1\right\}$$

If the cubic equation $X = 0$ has only one real root say α, let

$X = (x - \alpha)(x^2 - 2bx + c)$, $c - b^2 > 0$, $H^2 = \alpha^2 - 2\alpha b + c$

$$m = \frac{H - \alpha + b}{2H}, \qquad m_1 = \frac{H + \alpha - b}{2H}$$

$$\int_x^\infty \frac{dx}{X^{1/2}} = \frac{1}{H^{1/2}} \operatorname{cn}^{-1}\left(\frac{x - H - \alpha}{x + H - \alpha} \,\middle|\, m\right),$$

$$\int_{-\infty}^x \frac{dx}{(-X)^{1/2}} = \frac{1}{H^{1/2}} \operatorname{cn}^{-1}\left(\frac{\alpha - H - x}{\alpha + H - x} \,\middle|\, m_1\right),$$

$$\int_\alpha^x \frac{dx}{X^{1/2}} = \frac{1}{H^{1/2}} \operatorname{cn}^{-1}\left(\frac{H + \alpha - x}{H - \alpha + x} \,\middle|\, m\right),$$

$$\int_x^\alpha \frac{dx}{(-X)^{1/2}} = \frac{1}{H^{1/2}} \operatorname{cn}^{-1}\left(\frac{H - \alpha + x}{H + \alpha - x} \,\middle|\, m_1\right)$$

Elliptic Integrals of the Second Kind. Zeta Function

The elliptic integral of the second kind can be expressed in the form

$$\int^x \frac{t^2\, dt}{[(A_1 t^2 + B_1)(A_2 t^2 + B_2)]^{1/2}}$$

The various sign patterns of the radical are the same as those of the elliptic integral of the first kind, and the integral is reduced by the same substitutions. In this way the integral of the second kind can be brought to depend upon the function

$$E(u) = E(u \mid m) = \int_0^u \operatorname{dn}^2 (z \mid m)\, dz$$

and the *complete elliptic integral* of the second kind is $E(K) = E$. The function $E(u)$ has the property that $E(u + 2K) = E(u) + 2E$. It follows that the Jacobian *zeta function** defined by

$$Z(u) = Z(u \mid m) = E(u) - uE/K$$

is an odd periodic function of u with the period $2K$. It is not an elliptic function since it is not doubly periodic. By means of the table of $Z(u)$ elliptic integrals of the second kind are readily evaluated, see Numerical example 17.

Observe that $Z(u \mid 0) = 0$, $Z(u \mid 1) = \tanh u$. The function $Z(u)$ has the quasi addition theorem

$$Z(u + v) = Z(u) + Z(v) - m \operatorname{sn} u \operatorname{sn} v \operatorname{sn} (u + v)$$

This together with Jacobi's imaginary transformation namely

$$iZ(iu \mid m) = Z(u \mid m_1) + \pi u/(2KK') - \operatorname{dn} (u \mid m_1) \operatorname{sc} (u \mid m_1)$$

enables $Z(u)$ to be evaluated for complex arguments.

The addition theorem can also be used with advantage when interpolating across the table for intermediate values of m in those cases where u is large enough (greater than $\frac{1}{2}K$) to make the differences unmanageable. Thus

$$Z(u) = Z(u - K) - m \operatorname{sn} (u - K) \operatorname{sn} u,$$

$$Z(u) = Z(u - \tfrac{1}{2}K) + Z(\tfrac{1}{2}K) - m \operatorname{sn} (u - \tfrac{1}{2}K) \operatorname{sn} u \operatorname{sn} (\tfrac{1}{2}K)$$

In the table the sign attached to the second difference or given at the head of the column is the actual sign of the difference as found by giving the proper sign to the tabular entry to which that difference relates.

---

*The notation zn u is also used.

The Elliptic Integral of the Third Kind

This arises from the evaluation of integrals of the type

$$\int \frac{dt}{(1 + Ct^2)[(A_1t^2 + B_1)(A_2t^2 + B_2)]^{1/2}}$$

which, with the appropriate substitution indicated by the sign pattern of the radical, may be reduced to known functions together with

$$\int_0^u \frac{\operatorname{sn}^2 t}{1 + \nu \operatorname{sn}^2 t}\, dt$$

If we write $\nu = -m \operatorname{sn}^2 a$, this integral is clearly a constant multiple of

$$\Pi(u, a) = \int_0^u \frac{m \operatorname{sn} a \operatorname{cn} a \operatorname{dn} a \operatorname{sn}^2 t}{1 - m \operatorname{sn}^2 a \operatorname{sn}^2 t}\, dt$$

which is taken as the canonical form of the elliptic integral of the third kind. It can be evaluated in the form

$$\Pi(u, a) = u\, Z(a) + \frac{1}{2} \log_e \frac{\Theta(u - a)}{\Theta(u + a)},$$

where* $\Theta(u) = 1 - 2q \cos 2x + 2q^4 \cos 4x - \cdots, x = \pi u/(2K)$. It has the property

$$\Pi(u, a) - u\, Z(a) = \Pi(a, u) - a\, Z(u).$$

In practical applications a is usually imaginary and the evaluation of $\Theta(u)$ for imaginary values of the argument is necessary. For such cases $\Theta(u)$ does not lend itself readily to compact tabulation and must be found from the q series, which converges rapidly.

The *complete elliptic integral* of the third kind namely the integral whose upper limit is K is obtained at once as

$$\Pi(K, a) = KZ(a).$$

---

*The general term is $2(-1)^n q^{n^2} \cos 2n\, x$.

Short Table of E/K

m	E/K	E'/K'	m_1
0·1	0·9493 416	0·4285 242	0·9
0·2	·8972 125	·5221 013	0·8
0·3	·8433 234	·5982 907	0·7
0·4	·7872 725	·6660 082	0·6
0·5	·7284 733	·7284 733	0·5
m_1	E'/K'	E/K	m

Conformal Mapping

Rectangle on quarter-plane or half-plane

Determine the parameter m of elliptic functions from

$$\frac{K}{a} = \frac{K'}{b} = \lambda,$$

where a and b are given (see Numerical example 18).

Then the interior of the rectangle whose vertices are the points

$$(0, 0), (a, 0), (a, b), (0, b) \text{ in the } z\text{-plane } (z = x + iy)$$

is mapped on the quarter-plane $u > 0$, $v > 0$ in the w-plane ($w = u + iv$) by

$$w = \text{sn}\,(\lambda z \mid m)$$

The same transformation maps the interior of the rectangle whose vertices are $(\pm a, 0)$, $(\pm a, b)$ on the half-plane $v > 0$.

Exterior of a rectangle on half-plane

Let the vertices of the rectangle in the z-plane be the points $(\pm a, \pm b)$. Determine the parameter m of elliptic functions from

$$\frac{E - m_1 K}{b} = \frac{E' - mK'}{a}, \qquad b \leq a,$$

and let λ denote the value of either of these ratios. Then the region exterior to the rectangle is mapped on the half-plane $v > 0$ by

$$z = a - \frac{i}{\lambda}\left\{Z(\sigma \mid m) + \left(\frac{E}{K} - m_1\right)\sigma\right\},$$

$$w = \frac{1 - \operatorname{dn}(\sigma \mid m)}{m^{1/2} \operatorname{sn}(\sigma \mid m)},$$

where σ is an auxiliary variable.

Region exterior to two semi-infinite strips on half-plane

The region is that exterior to the semi-infinite strips

$$-a \leq x \leq a,\ y \geq b; \qquad -a \leq x \leq a,\ y \leq -b.$$

Determine m, K, E, from

$$\frac{b}{a} = \frac{(2 - m_1)K' - 2E'}{2(2E - m_1 K)}, \qquad \text{and let } \lambda = \frac{1}{2E - m_1 K}.$$

Then the above region is mapped on the half-plane $v < 0$ by

$$z = -b - \frac{ai\sigma}{K} - ai\lambda\left\{2Z(\sigma \mid m) + \frac{\operatorname{cn}(\sigma \mid m)\operatorname{dn}(\sigma \mid m)}{\operatorname{sn}(\sigma \mid m)}\right\},$$

$$w = \operatorname{ns}(\sigma \mid m).$$

Here σ is an auxiliary variable.

Isosceles right-angled triangle on half-plane

The interior of the triangle whose vertices are $(\pm 2^{-1/2}K, 0)$, $(0, 2^{-1/2}K)$, where $K = K(0\cdot5)$, is mapped on the half-plane $v > 0$ by

$$w = 2^{1/2} \operatorname{sn}(2^{1/2}z \mid 0\cdot5) \operatorname{dn}(2^{1/2}z \mid 0\cdot5).$$

Equilateral triangle on half-plane

The interior of the equilateral triangle whose vertices are $(0, 0)$, $(a, -3^{1/2}a)$, $(2a, 0)$ is mapped on the half-plane $v > 0$ by

$$w = \frac{\lambda\{1 + \mathrm{cn}\,(z \mid m)\}^2}{\mathrm{sn}\,(z \mid m)\,\mathrm{dn}\,(z \mid m)}, \qquad m = \sin^2 \frac{5\pi}{12}, \qquad \lambda = \frac{3^{1/4}}{6},$$

and a is determined by

$$\lambda(1 + \mathrm{cn}\,2a)^2 = \mathrm{sn}\,2a\,\mathrm{dn}\,2a$$

Half of an equilateral triangle on half-plane

The interior of the triangle whose vertices are $(0, 0)$, $(a, 0)$, $(0, 3^{1/2}a)$ is mapped on the half-plane $v > 0$ by

$$\frac{1 - w}{1 + w} = (1 - 3^{1/2}\,\mathrm{sc}^2\,z)^3, \qquad m = \sin^2 \frac{5\pi}{12},$$

and a is determined by $\mathrm{sc}^2\,a = 3^{-1/2}$.

Double half equilateral triangle on half-plane

The interior of the triangle whose vertices are $(0, 0)$, $(2(3^{1/2})\,a, 0)$, $(3^{1/2}a, a)$ is mapped on the half-plane $v > 0$ by

$$16\wp^3\left(\frac{z}{3(2^{1/3})}\,;\,0,\,1\right) = \frac{1}{w - w^2}.$$

and a is determined by

$$4\wp^3\left\{\frac{3^{1/2}a}{3(2^{1/3})}\,;\,0,\,1\right\} = 1$$

Rectangle on unit circle

Determine the parameter m of elliptic functions from

$$\frac{K}{2a} = \frac{K'}{2b} = \lambda.$$

Then the interior of the rectangle whose vertices are $(\pm a, \pm b)$ is mapped on the unit circle $|w| \leq 1$ by

$$w = \frac{\mathrm{sn}\,\lambda z\,\mathrm{dn}\,\lambda z}{\mathrm{cn}\,\lambda z}.$$

Observe that

$$w^2 = \frac{1 - \text{cn}\, 2\lambda\, z}{1 + \text{cn}\, 2\lambda\, z}.$$

For a square $m = 0 \cdot 5$.

Square on unit circle

Let $K = K(0 \cdot 5)$. Then the interior of the square whose vertices are $(\pm 2^{-1/2}K, 0)$, $(0, \pm 2^{-1/2}K)$ is mapped on the unit circle $|w| \leq 1$ by

$$2^{1/2}w = \text{sd}\,(2^{1/2}z \mid 0 \cdot 5).$$

Ellipse on unit circle

The interior of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is mapped on the unit circle $|w| \leq 1$ by

$$z = c \sin \lambda\sigma, \qquad w = m^{1/4}\, \text{sn}\,(\sigma \mid m),$$

where σ is an auxiliary variable, $c^2 = a^2 - b^2$, $\lambda = \pi/(2K)$, and m is determined by

$$q = \exp\,(-\pi K'/K) = \left(\frac{a - b}{a + b}\right)^2.$$

Factorisation of a Cubic Polynomial

A cubic equation with real coefficients has always one real root, which can be found by successive approximation or by the method indicated below. When a real root has been found the corresponding cubic polynomial can be resolved into the product of a linear and a quadratic factor.

The general cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$ can be reduced to one of the following forms by division by a and then substituting $x = y - b/a$

(i) $$y^3 + py \pm q = 0, \qquad p > 0$$

The only real root is

$$\mp \left(\frac{4p}{3}\right)^{1/2} \sinh \theta, \qquad \text{where } \sinh 3\theta = \left(\frac{27q^2}{4p^3}\right)^{1/2}.$$

(ii) $$y^3 - py \pm q = 0, \qquad p > 0$$

If $27q^2 > 4p^3$, the only real root is

$$\mp \left(\frac{4p}{3}\right)^{1/2} \cosh \theta, \qquad \text{where } \cosh 3\theta = \left(\frac{27q^2}{4p^3}\right)^{1/2}.$$

If $27q^2 < 4p^3$ there are three real roots namely

$$\mp \left(\frac{4p}{3}\right)^{1/2} \cos \theta, \qquad \mp \left(\frac{4p}{3}\right)^{1/2} \cos (\theta + 120°),$$

$$\mp \left(\frac{4p}{3}\right)^{1/2} \cos (\theta + 240°), \qquad \text{where } \cos 3\theta = \left(\frac{27q^2}{4p^3}\right)^{1/2}.$$

Factorisation of a Quartic Polynomial

Let the polynomial be $R = ax^4 + 4bx^3 + 6cx^2 + 4dx + e$ and let M and N be chosen such that

$$aR = (ax^2 + 2bx + c + 2\lambda a)^2 - (2Mx + N)^2$$

Comparing coefficients we find

$$M^2 = b^2 - ac + a^2\lambda, \qquad MN = bc - ad + 2ab\lambda,$$

$$N^2 = (c + 2a\lambda)^2 - ae$$

Elimination of M, N leads to the cubic equation

$$4a^3\lambda^3 - (ae - 4bd + 3c^2)a\lambda + ace$$

$$+ 2bcd - ad^2 - eb^2 - c^3 = 0.$$

Let μ be a real root of this cubic. We now find M from $M^2 = b^2 - ac + a^2\mu$, either sign may be taken for the square root, and then N by division of MN by M. In this way R is factorised in the form

$$R = \frac{1}{a}(ax^2 + 2bx + c + 2a\mu + 2Mx + N)$$

$$\cdot(ax^2 + 2bx + c + 2a\mu - 2Mx - N)$$

If all three roots of the cubic are real, the method furnishes three pairs of real quadratic factors. For applications to elliptic integrals the pairs should be so chosen that their zeros do not separate one another.

The Pendulum

The energy equation of a pendulum, simple or compound, can be expressed in the form

$$\tfrac{1}{2}l\dot{\theta}^2 - g\cos\theta = \tfrac{1}{2}l\omega^2 - g$$

where θ is the inclination to the downward vertical and ω is the value of $\dot{\theta}$ when $\theta = 0$. This leads to

(1) $$\dot{\theta}^2 = \omega^2(1 - m\sin^2\tfrac{1}{2}\theta), \qquad m = 4g/(l\omega^2)$$

If we take m as the parameter of Jacobian elliptic functions and write

(2) $$\sin\tfrac{1}{2}\theta = \operatorname{sn} u = \operatorname{sn}(u \mid m),$$ we have

(3) $$\cos\tfrac{1}{2}\theta = \operatorname{cn} u, \; \dot{\theta} = \omega \operatorname{dn} u$$

Since $d(\operatorname{sn} u)/dt = \operatorname{cn} u \operatorname{dn} u\, \dot{u}$, differentiation of (2) and the use of (3) leads to $\dot{u} = \frac{1}{2}\omega$ or $u = \frac{1}{2}\omega t$ if θ and u both vanish when $t = 0$. Thus

(4) $$\sin\tfrac{1}{2}\theta = \operatorname{sn}(\tfrac{1}{2}\omega t \mid m), \qquad m = 4g/(l\omega^2)$$

which solves the problem of the pendulum in terms of elliptic functions.*

Case (i). The pendulum makes a complete revolution in time T. In this case θ increases from 0 to π while t increases from 0 to $\frac{1}{2}T$, and therefore

$$\operatorname{sn}\left(\frac{1}{4}\omega T\right) = 1, \qquad \frac{1}{4}\omega T = K, \qquad T = \frac{4K}{\omega}, \qquad m = \frac{4g}{l\omega^2}$$

Case (ii). The pendulum oscillates through the angle α on each side of the vertical. Here $\dot{\theta} = 0$ when $\theta = \alpha$ and from (1)

$$m = \operatorname{cosec}^2 \tfrac{1}{2}\alpha, \qquad \tfrac{1}{2}\omega \operatorname{cosec} \tfrac{1}{2}\alpha = [(g/l)]^{1/2}$$

while from (4) $\quad \sin \frac{1}{2}\theta = \operatorname{sn}(\frac{1}{2}\omega t \mid \operatorname{cosec}^2 \frac{1}{2}\alpha)$

Since $m > 1$, we use Jacobi's real transformation (p. 19) to the reciprocal parameter to give

$$\sin \tfrac{1}{2}\theta = \sin \tfrac{1}{2}\alpha \operatorname{sn}(t(g/l)^{1/2} \mid \sin^2 \tfrac{1}{2}\alpha)$$

If T is the period, θ increases from 0 to α while t increases from 0 to $\frac{1}{4}T$ and therefore

$$\operatorname{sn}\left[\frac{1}{4}T\left(\frac{g}{l}\right)^{1/2}\right] = 1, \qquad T = 4K\left(\frac{l}{g}\right)^{1/2},$$

where $K = K(\sin^2 \frac{1}{2}\alpha)$ is obtained from the tables.

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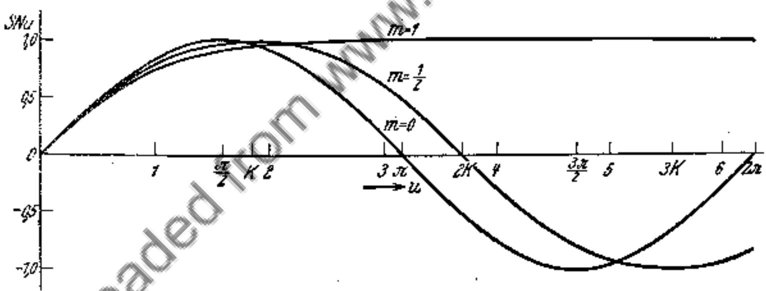
*Milne-Thomson, *Quarterly Journal of Mech. and Applied Math.* 2 (1949) p. 479.

Five-figure Table of the Elliptic Function sn $(u \mid m)$



m	0·0	0·1	0·2	0·3	0·4
u	sn u	sn u	sn u	sn u	sn u
0·00	00000	00000	00000	00000	00000
	1000	1000	1000	1000	1000
·01	01000	01000	01000	01000	01000
	1000	1000	1000	1000	1000
·02	02000	02000	02000	02000	02000
	1000	1000	999	999	999
·03	03000	03000	02999	02999	02999
	999	999	1000	1000	1000
·04	03999	03999	03999	03999	03999
	999	999	999	998	998
·05	04998	04998	04998	04997	04997
	998	998	998	998	998
·06	05996	05996	05996	05995	05995
	998	998	997	998	997
·07	06994	06994	06993	06993	06992
	997	997	997	996	996
·08	07991	07991	07990	07989	07988
	997	996	995	995	995
·09	08988	08987	08985	08984	08983
	995	995	995	994	994
·10	09983	09982	09980	09978	09977
	995	994	993	993	992
·11	10978	10976	10973	10971	10969
	993	992	993	992	991
·12	11971	11968	11966	11963	11960
	992	992	990	990	989
·13	12963	12960	12956	12953	12949
	991	990	989	988	987
·14	13954	13950	13945	13941	13936
	990	988	988	986	986
·15	14944	14938	14933	14927	14922
	988	987	985	985	983
·16	15932	15925	15918	15912	15905
	986	985	984	982	981
·17	16918	16910	16902	16894	16886
	985	983	982	980	979
·18	17903	17893	17884	17874	17865
	983	982	980	978	976
·19	18886	18875	18864	18852	18841
	981	979	977	976	974
·20	19867	19854	19841	19828	19815
	979	977	975	973	971
·21	20846	20831	20816	20801	20786
	977	975	973	971	968
·22	21823	21806	21789	21772	21754
	975	972	970	967	966
·23	22798	22778	22759	22739	22720
	972	970	967	965	962
·24	23770	23748	23726	23704	23682
	970	967	965	962	959
·25	24740	24715	24691	24666	24641
K	1·57080	1·61244	1·65962	1·71389	1·77752

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sn u	sn u	sn u	sn u	sn u	sn u
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01000	01000	01000	01000	01000	01000
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03998	03998	03998	03998	03998	03998
999	999	998	998	998	998
04997	04997	04996	04996	04996	04996
998	997	998	998	997	997
05995	05994	05994	05994	05993	05993
996	997	996	996	996	996
06991	06991	06990	06990	06989	06989
996	995	996	995	995	994
07987	07986	07986	07985	07984	07983
995	995	993	993	993	993
08982	08981	08979	08978	08977	08976
993	992	993	992	991	991
09975	09973	09972	09970	09968	09967
992	992	990	990	990	989
10967	10965	10962	10960	10958	10956
990	989	989	988	988	987
11957	11954	11951	11948	11946	11943
988	988	987	986	985	984
12945	12942	12938	12934	12931	12927
987	985	985	984	983	982
13932	13927	13923	13918	13914	13909
984	984	982	982	980	980
14916	14911	14905	14900	14894	14889
982	981	980	978	978	976
15898	15892	15885	15878	15872	15865
980	978	977	976	974	973
16878	16870	16862	16854	16846	16838
977	976	975	973	972	970
17855	17846	17837	17827	17818	17808
975	973	971	970	968	967
18830	18819	18808	18797	18786	18775
972	970	968	966	964	963
19802	19789	19776	19763	19750	19738
969	967	965	963	962	959
20771	20756	20741	20726	20712	20697
966	964	962	960	957	955
21737	21720	21703	21686	21669	21652
963	961	958	956	953	951
22700	22681	22661	22642	22622	22603
960	957	955	952	950	947
23660	23638	23616	23594	23572	23550
956	953	950	947	945	942
24616	24591	24566	24541	24517	24492
1·85407	1·94957	2·07536	2·25721	2·57809	

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u	sn u	sn u	sn u	sn u	sn u
0·25	24740	24715	24691	24666	24641
	968	965	961	958	955
·26	25708	25680	25652	25624	25596
	965	962	959	956	953
·27	26673	26642	26611	26580	26549
	963	959	955	952	948
·28	27636	27601	27566	27532	27497
	959	956	953	948	945
·29	28595	28557	28519	28480	28442
	957	953	949	945	941
·30	29552	29510	29468	29425	29383
	954	949	945	942	938
·31	30506	30459	30413	30367	30321
	951	947	942	937	933
·32	31457	31406	31355	31304	31254
	947	943	939	934	929
·33	32404	32349	32294	32238	32183
	945	939	934	930	925
·34	33349	33288	33228	33168	33108
	941	936	931	926	920
·35	34290	34224	34159	34094	34028
	937	933	927	921	916
·36	35227	35157	35086	35015	34944
	935	928	923	917	912
·37	36162	36085	36009	35932	35856
	930	925	918	913	907
·38	37092	37010	36927	36845	36763
	927	920	915	908	902
·39	38019	37930	37842	37753	37665
	923	917	910	904	897
·40	38942	38847	38752	38657	38562
	919	912	905	899	892
·41	39861	39759	39657	39556	39454
	915	908	902	894	888
·42	40776	40667	40559	40450	40342
	911	904	896	889	882
·43	41687	41571	41455	41339	41224
	907	899	892	885	877
·44	42594	42470	42347	42224	42101
	903	895	887	879	871
·45	43497	43365	43234	43103	42972
	898	891	882	874	867
·46	44395	44256	44116	43977	43839
	894	885	878	869	860
·47	45289	45141	44994	44846	44699
	889	881	872	864	856
·48	46178	46022	45866	45710	45555
	885	876	867	859	849
·49	47063	46898	46733	46569	46404
	880	871	862	853	844
·50	47943	47769	47595	47422	47248
K	1·57080	1·61244	1·65962	1·71389	1·77752

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sn u		sn u		sn u		sn u		sn u		sn u	
24676	953	24591	950	24566	947	24541	944	24517	940	24492	938
25569	949	25541	946	25513	942	25485	939	25457	936	25430	932
26518	945	26487	941	26455	939	26424	935	26393	932	26362	929
27463	941	27428	938	27394	934	27359	931	27325	927	27291	922
28404	937	28366	933	28328	929	28290	925	28252	921	28213	918
29341	933	29299	929	29257	925	29215	921	29173	917	29131	913
30274	929	30228	925	30182	920	30136	916	30090	911	30044	907
31203	925	31153	919	31102	915	31052	910	31001	906	30951	901
32128	920	32072	916	32017	911	31962	906	31907	901	31852	896
33048	915	32988	910	32928	905	32868	900	32808	895	32748	890
33963	911	33898	905	33833	900	33768	894	33703	889	33638	883
34874	906	34803	900	34733	894	34662	889	34592	883	34521	878
35779	901	35703	895	35627	889	35551	883	35475	878	35399	872
36680	896	36598	890	36516	884	36434	878	36353	871	36271	865
37576	891	37488	885	37400	878	37312	872	37224	865	37136	859
38467	886	38373	879	38278	872	38184	865	38089	859	37995	852
39353	880	39252	873	39150	867	39049	860	38948	853	38847	846
40233	875	40125	868	40017	860	39909	853	39801	846	39693	839
41108	870	40993	862	40877	855	40762	847	40647	840	40532	832
41978	864	41855	856	41732	849	41609	841	41487	833	41364	826
42842	858	42711	850	42581	842	42450	835	42320	826	42190	818
43700	852	43561	845	43423	836	43285	827	43146	820	43008	812
44552	847	44406	838	44259	830	44112	822	43966	813	43820	804
45399	841	45244	832	45089	823	44934	815	44779	806	44624	798
46240	835	46076	826	45912	817	45749	808	45585	799	45422	790
47075		46902		46729		46557		46384		46212	
1·85407		1·94957		2·07536		2·25721		2·57809			

m	0·0	0·1	0·2	0·3	0·4
u	sn u	sn u	sn u	sn u	sn u
0·50	47943_{875}	47769_{866}	47595_{857}	47422_{847}	47248_{838}
·51	48818_{870}	48635_{860}	48452_{851}	48269_{842}	48086_{833}
·52	49688_{865}	49495_{856}	49303_{846}	49111_{836}	48919_{826}
·53	50553_{861}	50351_{850}	50149_{840}	49947_{830}	49745_{821}
·54	51414_{855}	51201_{845}	50989_{835}	50777_{825}	50566_{814}
·55	52269_{850}	52046_{840}	51824_{829}	51602_{819}	51380_{808}
·56	53119_{844}	52886_{834}	52653_{823}	52421_{812}	52188_{802}
·57	53963_{839}	53720_{828}	53476_{818}	53233_{807}	52990_{796}
·58	54802_{834}	54548_{823}	54294_{812}	54040_{801}	53786_{790}
·59	55636_{828}	55371_{817}	55106_{806}	54841_{794}	54576_{783}
·60	56464_{823}	56188_{811}	55912_{799}	55635_{789}	55359_{777}
·61	57287_{817}	56999_{805}	56711_{794}	56424_{782}	56136_{770}
·62	58104_{810}	57804_{799}	57505_{787}	57206_{775}	56906_{764}
·63	58914_{806}	58603_{794}	58292_{782}	57981_{770}	57670_{758}
·64	59720_{799}	59397_{787}	59074_{775}	58751_{763}	58428_{751}
·65	60519_{793}	60184_{781}	59849_{769}	59514_{756}	59179_{744}
·66	61312_{787}	60965_{774}	60618_{762}	60270_{750}	59923_{737}
·67	62099_{780}	61739_{769}	61380_{756}	61020_{744}	60660_{731}
·68	62879_{775}	62508_{762}	62136_{749}	61764_{736}	61391_{724}
·69	63654_{768}	63270_{755}	62885_{743}	62500_{730}	62115_{717}
·70	64422_{761}	64025_{749}	63628_{736}	63230_{724}	62832_{711}
·71	65183_{755}	64774_{743}	64364_{730}	63954_{717}	63543_{704}
·72	65938_{749}	65517_{735}	65094_{723}	64671_{709}	64247_{696}
·73	66687_{742}	66252_{729}	65817_{716}	65380_{704}	64943_{690}
·74	67429_{735}	66981_{723}	66533_{709}	66084_{696}	65633_{683}
·75	68164	67704	67242	66780	66316
K	1·57080	1·61244	1·65962	1·71389	1·77752

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sn *u*	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
47075_{829}	46902_{820}	46729_{811}	46557_{801}	46384_{792}	46212_{783}
47904_{823}	47722_{813}	47540_{804}	47358_{794}	47176_{785}	46995_{775}
48727_{817}	48535_{807}	48344_{797}	48152_{788}	47961_{778}	47770_{768}
49544_{810}	49342_{801}	49141_{790}	48940_{780}	48739_{770}	48538_{761}
50354_{804}	50143_{794}	49931_{784}	49720_{774}	49509_{764}	49299_{753}
51158_{798}	50937_{787}	50715_{777}	50494_{767}	50273_{756}	50052_{746}
51956_{792}	51724_{781}	51492_{770}	51261_{759}	51029_{749}	50798_{738}
52748_{785}	52505_{774}	52262_{764}	52020_{752}	51778_{741}	51536_{731}
53533_{778}	53279_{768}	53026_{756}	52772_{746}	52519_{735}	52267_{723}
54311_{772}	54047_{760}	53782_{749}	53518_{738}	53254_{726}	52990_{715}
55083_{766}	54807_{754}	54531_{743}	54256_{731}	53980_{720}	53705_{708}
55849_{758}	55561_{747}	55274_{735}	54987_{723}	54700_{711}	54413_{700}
56607_{752}	56308_{740}	56009_{729}	55710_{717}	55411_{705}	55113_{692}
57359_{746}	57048_{734}	56738_{721}	56427_{709}	56116_{697}	55805_{685}
58105_{738}	57782_{726}	57459_{714}	57136_{702}	56813_{689}	56490_{677}
58843_{732}	58508_{720}	58173_{707}	57838_{694}	57502_{682}	57167_{669}
59575_{725}	59228_{712}	58880_{700}	58532_{687}	58184_{675}	57836_{662}
60300_{718}	59940_{705}	59580_{692}	59219_{680}	58859_{667}	58498_{654}
61018_{712}	60645_{699}	60272_{686}	59899_{672}	59526_{659}	59152_{646}
61730_{704}	61344_{691}	60958_{678}	60571_{666}	60185_{652}	59798_{639}
62434_{697}	62035_{684}	61636_{671}	61237_{657}	60837_{644}	60437_{631}
63131_{691}	62719_{678}	62307_{664}	61894_{651}	61481_{637}	61068_{623}
63822_{683}	63397_{670}	62971_{657}	62545_{643}	62118_{629}	61691_{616}
64505_{677}	64067_{663}	63628_{649}	63188_{636}	62747_{622}	62307_{608}
65182_{669}	64730_{656}	64277_{642}	63824_{628}	63369_{615}	62915_{600}
65851	65386	64919	64452	63984	63515
1·85407	1·94957	2·07536	2·25721	2·57809	

0·75 – 1·00

m	0·0	0·1	0·2	0·3	0·4
u	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
0·75	68164_{728}	67704_{715}	67242_{703}	66780_{689}	66316_{676}
·76	68892_{722}	68419_{709}	67945_{695}	67469_{682}	66992_{669}
·77	69614_{714}	69128_{701}	68640_{689}	68151_{676}	67661_{662}
·78	70328_{707}	69829_{695}	69329_{682}	68827_{668}	68323_{655}
·79	71035_{701}	70524_{688}	70011_{674}	69495_{662}	68978_{648}
·80	71736_{693}	71212_{680}	70685_{668}	70157_{654}	69626_{641}
·81	72429_{686}	71892_{673}	71353_{660}	70811_{647}	70267_{634}
·82	73115_{678}	72565_{666}	72013_{653}	71458_{641}	70901_{627}
·83	73793_{671}	73231_{659}	72666_{647}	72099_{633}	71528_{619}
·84	74464_{664}	73890_{652}	73313_{638}	72732_{626}	72147_{613}
·85	75128_{656}	74542_{644}	73951_{632}	73358_{618}	72760_{605}
·86	75784_{649}	75186_{637}	74583_{624}	73976_{612}	73365_{599}
·87	76433_{641}	75823_{629}	75207_{617}	74588_{604}	73964_{591}
·88	77074_{633}	76452_{621}	75824_{610}	75192_{597}	74555_{584}
·89	77707_{626}	77073_{615}	76434_{602}	75789_{590}	75139_{577}
·90	78333_{617}	77688_{606}	77036_{595}	76379_{583}	75716_{570}
·91	78950_{610}	78294_{599}	77631_{588}	76962_{575}	76286_{563}
·92	79560_{602}	78893_{591}	78219_{580}	77537_{568}	76849_{556}
·93	80162_{594}	79484_{584}	78799_{572}	78105_{561}	77405_{548}
·94	80756_{586}	80068_{575}	79371_{565}	78666_{554}	77953_{541}
·95	81342_{577}	80643_{568}	79936_{557}	79220_{546}	78494_{535}
·96	81919_{570}	81211_{560}	80493_{550}	79766_{539}	79029_{527}
·97	82489_{561}	81771_{552}	81043_{542}	80305_{531}	79556_{520}
·98	83050_{553}	82323_{544}	81585_{535}	80836_{524}	80076_{513}
·99	83603_{544}	82867_{537}	82120_{527}	81360_{517}	80589_{506}
1·00	84147	83404	82647	81877	81095
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
sn u	sn u	sn u	sn u	sn u	sn u
65851_{663}	65386_{649}	64919_{635}	64452_{621}	63984_{607}	63515_{593}
66514_{655}	66035_{641}	65554_{628}	65073_{614}	64591_{599}	64108_{585}
67169_{649}	66676_{635}	66182_{621}	65687_{606}	65190_{593}	64693_{578}
67818_{641}	67311_{628}	66803_{613}	66293_{600}	65783_{584}	65271_{570}
68459_{634}	67939_{620}	67416_{607}	66893_{592}	66367_{578}	65841_{563}
69093_{628}	68559_{613}	68023_{599}	67485_{584}	66945_{570}	66404_{555}
69721_{620}	69172_{606}	68622_{592}	68069_{578}	67515_{563}	66959_{548}
70341_{613}	69778_{600}	69214_{585}	68647_{570}	68078_{556}	67507_{541}
70954_{606}	70378_{592}	69799_{577}	69217_{563}	68634_{548}	68048_{533}
71560_{599}	70970_{585}	70376_{571}	69780_{557}	69182_{541}	68581_{526}
72159_{592}	71555_{578}	70947_{564}	70337_{549}	69723_{534}	69107_{519}
72751_{585}	72133_{570}	71511_{556}	70886_{542}	70257_{527}	69626_{511}
73336_{577}	72703_{564}	72067_{550}	71428_{534}	70784_{520}	70137_{505}
73913_{571}	73267_{557}	72617_{542}	71962_{528}	71304_{513}	70642_{497}
74484_{564}	73824_{550}	73159_{536}	72490_{521}	71817_{506}	71139_{491}
75048_{556}	74374_{543}	73695_{529}	73011_{514}	72323_{499}	71630_{483}
75604_{550}	74917_{536}	74224_{522}	73525_{508}	72822_{492}	72113_{477}
76154_{543}	75453_{529}	74746_{515}	74033_{500}	73314_{485}	72590_{469}
76697_{535}	75982_{522}	75261_{508}	74533_{493}	73799_{479}	73059_{463}
77232_{529}	76504_{515}	75769_{501}	75026_{487}	74278_{471}	73522_{456}
77761_{522}	77019_{509}	76270_{495}	75513_{480}	74749_{465}	73978_{450}
78283_{514}	77528_{502}	76765_{488}	75993_{474}	75214_{459}	74428_{442}
78797_{508}	78030_{494}	77253_{481}	76467_{467}	75673_{451}	74870_{437}
79305_{501}	78524_{488}	77734_{474}	76934_{460}	76124_{446}	75307_{429}
79806_{494}	79012_{482}	78208_{468}	77394_{454}	76570_{439}	75736_{423}
80300	79494	78676	77848	77009	76159
1·85407	1·94957	2·07536	2·25721	2·57809	

1·00 – 1·25

m	0·0	0·1	0·2	0·3	0·4
u	sn u	sn u	sn u	sn u	sn u
1·00	84147_{536}	83404_{528}	82647_{519}	81877_{510}	81095_{498}
1·01	84683_{528}	83932_{520}	83166_{512}	82387_{502}	81593_{492}
1·02	85211_{519}	84452_{512}	83678_{504}	82889_{494}	82085_{485}
1·03	85730_{510}	84964_{504}	84182_{496}	83383_{488}	82570_{477}
1·04	86240_{502}	85468_{496}	84678_{488}	83871_{480}	83047_{471}
1·05	86742_{494}	85964_{487}	85166_{481}	84351_{473}	83518_{463}
1·06	87236_{484}	86451_{480}	85647_{473}	84824_{465}	83981_{457}
1·07	87720_{476}	86931_{471}	86120_{465}	85289_{458}	84438_{449}
1·08	88196_{467}	87402_{462}	86585_{457}	85747_{450}	84887_{443}
1·09	88663_{458}	87864_{455}	87042_{450}	86197_{444}	85330_{435}
1·10	89121_{449}	88319_{446}	87492_{442}	86641_{435}	85765_{429}
1·11	89570_{440}	88765_{438}	87934_{434}	87076_{429}	86194_{421}
1·12	90010_{431}	89203_{429}	88368_{426}	87505_{421}	86615_{415}
1·13	90441_{422}	89632_{421}	88794_{418}	87926_{414}	87030_{408}
1·14	90863_{413}	90053_{413}	89212_{410}	88340_{406}	87438_{401}
1·15	91276_{404}	90466_{404}	89622_{403}	88746_{399}	87839_{394}
1·16	91680_{395}	90870_{396}	90025_{394}	89145_{392}	88233_{387}
1·17	92075_{386}	91266_{387}	90419_{387}	89537_{385}	88620_{380}
1·18	92461_{376}	91653_{378}	90806_{379}	89922_{377}	89000_{373}
1·19	92837_{367}	92031_{370}	91185_{371}	90299_{369}	89373_{367}
1·20	93204_{358}	92401_{362}	91556_{363}	90668_{363}	89740_{360}
1·21	93562_{348}	92763_{352}	91919_{355}	91031_{355}	90100_{353}
1·22	93910_{339}	93115_{344}	92274_{347}	91386_{348}	90453_{346}
1·23	94249_{329}	93459_{336}	92621_{339}	91734_{340}	90799_{340}
1·24	94578_{320}	93795_{327}	92960_{331}	92074_{334}	91139_{333}
1·25	94898	94122	93291	92408	91472
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
sn u	sn u	sn u	sn u	sn u	sn u
80300$_{487}$	79494$_{475}$	78676$_{462}$	77848$_{447}$	77000$_{441}$	76159$_{417}$
80787$_{481}$	79969$_{468}$	79138$_{455}$	78295$_{441}$	77441$_{426}$	76576$_{411}$
81268$_{473}$	80437$_{461}$	79593$_{448}$	78736$_{434}$	77867$_{420}$	76987$_{404}$
81741$_{467}$	80898$_{455}$	80041$_{442}$	79170$_{429}$	78287$_{413}$	77391$_{398}$
82208$_{460}$	81353$_{448}$	80483$_{436}$	79599$_{422}$	78700$_{408}$	77789$_{392}$
82668$_{453}$	81801$_{442}$	80919$_{429}$	80021$_{415}$	79108$_{401}$	78181$_{385}$
83121$_{446}$	82243$_{435}$	81348$_{423}$	80436$_{410}$	79509$_{395}$	78566$_{380}$
83567$_{440}$	82678$_{429}$	81771$_{416}$	80846$_{403}$	79904$_{389}$	78946$_{374}$
84007$_{433}$	83107$_{422}$	82187$_{411}$	81249$_{398}$	80293$_{383}$	79320$_{368}$
84440$_{427}$	83529$_{416}$	82598$_{404}$	81647$_{391}$	80676$_{378}$	79688$_{362}$
84867$_{419}$	83945$_{410}$	83002$_{399}$	82038$_{386}$	81054$_{371}$	80050$_{356}$
85286$_{414}$	84355$_{403}$	83401$_{392}$	82424$_{379}$	81425$_{366}$	80406$_{351}$
85700$_{406}$	84758$_{398}$	83793$_{386}$	82803$_{374}$	81791$_{360}$	80757$_{345}$
86106$_{400}$	85156$_{391}$	84179$_{380}$	83177$_{368}$	82151$_{355}$	81102$_{339}$
86506$_{394}$	85547$_{384}$	84559$_{375}$	83545$_{363}$	82506$_{349}$	81441$_{334}$
86900$_{387}$	85931$_{379}$	84934$_{368}$	83908$_{356}$	82855$_{343}$	81775$_{329}$
87287$_{381}$	86310$_{373}$	85302$_{363}$	84264$_{352}$	83198$_{338}$	82104$_{323}$
87668$_{374}$	86683$_{366}$	85665$_{357}$	84616$_{345}$	83536$_{333}$	82427$_{318}$
88042$_{368}$	87049$_{360}$	86022$_{351}$	84961$_{340}$	83869$_{327}$	82745$_{313}$
88410$_{362}$	87409$_{355}$	86373$_{345}$	85301$_{335}$	84196$_{322}$	83058$_{307}$
88772$_{355}$	87764$_{349}$	86718$_{340}$	85636$_{329}$	84518$_{317}$	83365$_{303}$
89127$_{349}$	88113$_{342}$	87058$_{335}$	85965$_{324}$	84835$_{311}$	83668$_{297}$
89476$_{342}$	88455$_{337}$	87393$_{328}$	86289$_{319}$	85146$_{307}$	83965$_{293}$
89818$_{337}$	88792$_{331}$	87721$_{324}$	86608$_{313}$	85453$_{302}$	84258$_{288}$
90155$_{330}$	89123$_{325}$	88045$_{318}$	86921$_{309}$	85755$_{296}$	84546$_{282}$
90485	89448	88363	87230	86051	84828
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
1·25	94898_{311}	94122_{318}	93291_{324}	92408_{326}	91472_{326}
1·26	95209_{301}	94440_{309}	93615_{315}	92734_{318}	91798_{320}
1·27	95510_{292}	94749_{301}	93930_{307}	93052_{312}	92118_{313}
1·28	95802_{282}	95050_{292}	94237_{300}	93364_{304}	92431_{306}
1·29	96084_{272}	95342_{283}	94537_{291}	93668_{297}	92737_{300}
1·30	96356_{262}	95625_{275}	94828_{284}	93965_{290}	93037_{293}
1·31	96618_{254}	95900_{265}	95112_{275}	94255_{282}	93330_{287}
1·32	96872_{243}	96165_{257}	95387_{268}	94537_{276}	93617_{280}
1·33	97115_{233}	96422_{248}	95655_{259}	94813_{268}	93897_{273}
1·34	97348_{224}	96670_{240}	95914_{252}	95081_{261}	94170_{267}
1·35	97572_{214}	96910_{230}	96166_{243}	95342_{253}	94437_{261}
1·36	97786_{205}	97140_{222}	96409_{236}	95595_{247}	94698_{254}
1·37	97991_{194}	97362_{212}	96645_{228}	95842_{239}	94952_{247}
1·38	98185_{185}	97574_{204}	96873_{219}	96081_{232}	95199_{242}
1·39	98370_{175}	97778_{195}	97092_{212}	96313_{225}	95441_{234}
1·40	98545_{165}	97973_{186}	97304_{204}	96538_{218}	95675_{229}
1·41	98710_{155}	98159_{177}	97508_{195}	96756_{211}	95904_{222}
1·42	98865_{145}	98336_{168}	97703_{188}	96967_{203}	96126_{216}
1·43	99010_{136}	98504_{160}	97891_{180}	97170_{197}	96342_{209}
1·44	99146_{125}	98664_{150}	98071_{171}	97367_{189}	96551_{203}
1·45	99271_{116}	98814_{141}	98242_{164}	97556_{182}	96754_{197}
1·46	99387_{105}	98955_{133}	98406_{156}	97738_{175}	96951_{190}
1·47	99492_{96}	99088_{123}	98562_{147}	97913_{168}	97141_{184}
1·48	99588_{86}	99211_{115}	98709_{140}	98081_{161}	97325_{178}
1·49	99674_{75}	99326_{105}	98849_{132}	98242_{154}	97503_{172}
1·50	99749	99431	98981	98396	97675
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
sn *u*	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
90485_{324}	89448_{320}	88363_{312}	87230_{303}	86051_{292}	84828_{278}
90809_{318}	89768_{314}	88675_{308}	87533_{299}	86343_{287}	85106_{274}
91127_{312}	90082_{308}	88983_{302}	87832_{293}	86630_{282}	85380_{268}
91439_{306}	90390_{303}	89285_{296}	88125_{288}	86912_{278}	85648_{265}
91745_{300}	90693_{297}	89581_{292}	88413_{284}	87190_{273}	85913_{259}
92045_{293}	90990_{291}	89873_{287}	88697_{279}	87463_{268}	86172_{256}
92338_{288}	91281_{286}	90160_{281}	88976_{274}	87731_{264}	86428_{250}
92626_{282}	91567_{281}	90441_{277}	89250_{269}	87995_{259}	86678_{247}
92908_{276}	91848_{275}	90718_{271}	89519_{265}	88254_{255}	86925_{242}
93184_{270}	92123_{270}	90989_{267}	89784_{260}	88509_{251}	87167_{238}
93454_{264}	92393_{264}	91256_{261}	90044_{255}	88760_{246}	87405_{234}
93718_{258}	92657_{260}	91517_{257}	90299_{251}	89006_{242}	87639_{230}
93976_{253}	92917_{254}	91774_{252}	90550_{247}	89248_{238}	87869_{226}
94229_{247}	93171_{249}	92026_{247}	90797_{243}	89486_{234}	88095_{222}
94476_{241}	93420_{243}	92273_{243}	91040_{238}	89720_{230}	88317_{218}
94717_{235}	93663_{239}	92516_{238}	91278_{233}	89950_{225}	88535_{214}
94952_{230}	93902_{233}	92754_{234}	91511_{230}	90175_{222}	88749_{211}
95182_{224}	94135_{229}	92988_{228}	91741_{225}	90397_{218}	88960_{207}
95406_{218}	94364_{223}	93216_{225}	91966_{221}	90615_{215}	89167_{203}
95624_{213}	94587_{218}	93441_{220}	92187_{218}	90830_{210}	89370_{199}
95837_{207}	94805_{214}	93661_{215}	92405_{213}	91040_{207}	89569_{196}
96044_{202}	95019_{208}	93876_{211}	92618_{209}	91247_{203}	89765_{193}
96246_{196}	95227_{204}	94087_{207}	92827_{205}	91450_{199}	89958_{189}
96442_{190}	95431_{199}	94294_{202}	93032_{202}	91649_{196}	90147_{185}
96632_{186}	95630_{194}	94496_{199}	93234_{198}	91845_{192}	90332_{183}
96818	95824	94695	93432	92037	90515
1·85407	**1·94957**	**2·07536**	**2·25721**	**2·57809**	

m	0·0	0·1	0·2	0·3	0·4
u	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
1·50	99749_{66}	99431_{97}	98981_{124}	98396_{147}	97675_{165}
1·51	99815_{56}	99528_{88}	99105_{115}	98543_{139}	97840_{160}
1·52	99871_{46}	99616_{78}	99220_{108}	98682_{133}	98000_{153}
1·53	99917_{36}	99694_{70}	99328_{100}	98815_{125}	98153_{147}
1·54	99953_{25}	99764_{61}	99428_{91}	98940_{119}	98300_{140}
1·55	99978_{16}	99825_{51}	99519_{84}	99059_{111}	98440_{135}
1·56	99994_{6}	99876_{43}	99603_{76}	99170_{105}	98575_{128}
1·57	$1·00000_{4}$	99919_{34}	99679_{67}	99275_{97}	98703_{123}
1·58	99996_{14}	99953_{24}	99746_{60}	99372_{90}	98826_{116}
1·59	99982_{25}	99977_{16}	99806_{52}	99462_{84}	98942_{110}
1·60	99957_{34}	99993_{7}	99858_{44}	99546_{76}	99052_{104}
1·61	99923_{44}	$1·00000_{3}$	99902_{35}	99622_{69}	99156_{98}
1·62	99879_{54}	99997_{11}	99937_{28}	99691_{63}	99254_{92}
1·63	99825_{64}	99986_{20}	99965_{20}	99754_{55}	99346_{86}
1·64	99761_{74}	99966_{29}	99985_{11}	99809_{48}	99432_{80}
1·65	99687_{85}	99937_{39}	99996_{4}	99857_{41}	99512_{73}
1·66	99602_{94}	99898_{47}	$1·00000_{4}$	99898_{35}	99585_{68}
1·67	99508_{104}	99851_{56}	99996_{13}	99933_{27}	99653_{61}
1·68	99404_{114}	99795_{66}	99983_{20}	99960_{20}	99714_{56}
1·69	99290_{124}	99729_{74}	99963_{28}	99980_{13}	99770_{50}
1·70	99166_{133}	99655_{83}	99935_{37}	99993_{6}	99820_{43}
1·71	99033_{144}	99572_{92}	99898_{44}	99999	99863_{38}
1·72	98889_{154}	99480_{102}	99854_{52}	99999_{8}	99901_{31}
1·73	98735_{163}	99378_{110}	99802_{60}	99991_{15}	99932_{26}
1·74	98572_{173}	99268_{119}	99742_{69}	99976_{22}	99958_{19}
1·75	98399	99149	99673	99954	99977
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
sn u	sn u	sn u	sn u	sn u	sn u
96818_{179}	95824_{189}	94695_{194}	93432_{194}	92037_{189}	90515_{179}
96997_{175}	96013_{185}	94889_{189}	93626_{190}	92226_{186}	90694_{176}
97172_{168}	96198_{179}	95078_{186}	93816_{186}	92412_{182}	90870_{172}
97340_{164}	96377_{176}	95264_{182}	94002_{183}	92594_{179}	91042_{170}
97504_{158}	96553_{170}	95446_{177}	94185_{179}	92773_{175}	91212_{167}
97662_{153}	96723_{166}	95623_{174}	94364_{176}	92948_{173}	91379_{163}
97815_{147}	96889_{161}	95797_{169}	94540_{172}	93121_{169}	91542_{161}
97962_{143}	97050_{157}	95966_{166}	94712_{169}	93290_{166}	91703_{157}
98105_{137}	97207_{152}	96132_{162}	94881_{165}	93456_{163}	91860_{155}
98242_{131}	97359_{148}	96294_{158}	95046_{162}	93619_{160}	92015_{152}
98373_{127}	97507_{143}	96452_{154}	95208_{159}	93779_{157}	92167_{149}
98500_{121}	97650_{139}	96606_{150}	95367_{156}	93936_{154}	92316_{146}
98621_{116}	97789_{134}	96756_{146}	95523_{152}	94090_{151}	92462_{144}
98737_{111}	97923_{130}	96902_{143}	95675_{149}	94241_{149}	92606_{141}
98848_{105}	98053_{126}	97045_{139}	95824_{145}	94390_{145}	92747_{139}
98953_{101}	98179_{121}	97184_{135}	95969_{143}	94535_{143}	92886_{136}
99054_{95}	98300_{117}	97319_{132}	96112_{139}	94678_{140}	93022_{133}
99149_{91}	98417_{112}	97451_{128}	96251_{137}	94818_{138}	93155_{131}
99240_{85}	98529_{108}	97579_{125}	96388_{133}	94956_{135}	93286_{129}
99325_{80}	98637_{104}	97704_{121}	96521_{131}	95091_{132}	93415_{126}
99405_{75}	98741_{100}	97825_{117}	96652_{127}	95223_{130}	93541_{124}
99480_{70}	98841_{96}	97942_{114}	96779_{125}	95353_{127}	93665_{121}
99550_{64}	98937_{91}	98056_{111}	96904_{122}	95480_{125}	93786_{120}
99614_{60}	99028_{87}	98167_{107}	97026_{119}	95605_{122}	93906_{117}
99674_{55}	99115_{83}	98274_{103}	97145_{116}	95727_{120}	94023_{115}
99729	99198	98377	97261	95847	94138
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
1·75	98399_{184}	99149_{128}	99673_{76}	99954_{28}	99977_{14}
1·76	98215_{193}	99021_{137}	99597_{84}	99926_{36}	99991_{7}
1·77	98022_{202}	98884_{146}	99513_{93}	99890_{43}	99998_{2}
1·78	97820_{213}	98738_{155}	99420_{100}	99847_{50}	$1·00000_{5}$
1·79	97607_{222}	98583_{164}	99320_{108}	99797_{57}	99995_{10}
1·80	97385_{232}	98419_{172}	99212_{117}	99740_{63}	99985_{17}
1·81	97153_{242}	98247_{182}	99095_{124}	99677_{71}	99968_{22}
1·82	96911_{252}	98065_{191}	98971_{132}	99606_{78}	99946_{29}
1·83	96659_{261}	97874_{199}	98839_{140}	99528_{85}	99917_{34}
1·84	96398_{270}	97675_{209}	98699_{149}	99443_{92}	99883_{41}
1·85	96128_{281}	97466_{217}	98550_{156}	99351_{99}	99842_{46}
1·86	95847_{290}	97249_{226}	98394_{164}	99252_{106}	99796_{53}
1·87	95557_{299}	97023_{235}	98230_{173}	99146_{113}	99743_{58}
1·88	95258_{309}	96788_{243}	98057_{180}	99033_{120}	99685_{65}
1·89	94949_{319}	96545_{253}	97877_{188}	98913_{127}	99620_{71}
1·90	94630_{328}	96292_{261}	97689_{196}	98786_{134}	99549_{76}
1·91	94302_{337}	96031_{271}	97493_{205}	98652_{141}	99473_{83}
1·92	93965_{347}	95760_{279}	97288_{212}	98511_{149}	99390_{89}
1·93	93618_{356}	95481_{287}	97076_{220}	98362_{155}	99301_{95}
1·94	93262_{366}	95194_{297}	96856_{228}	98207_{163}	99206_{101}
1·95	92896_{375}	94897_{305}	96628_{237}	98044_{169}	99105_{107}
1·96	92521_{384}	94592_{314}	96391_{244}	97875_{177}	98998_{113}
1·97	92137_{393}	94278_{322}	96147_{252}	97698_{184}	98885_{119}
1·98	91744_{403}	93956_{332}	95895_{260}	97514_{190}	98766_{125}
1·99	91341_{411}	93624_{340}	95635_{268}	97324_{198}	98641_{132}
2·00	90930	93284	95367	97126	98509
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
sn u	sn u	sn u	sn u	sn u	sn u
99729$_{49}$	99198$_{79}$	98377$_{100}$	97261$_{113}$	95847$_{117}$	94138$_{112}$
99778$_{45}$	99277$_{75}$	98477$_{97}$	97374$_{110}$	95964$_{116}$	94250$_{111}$
99823$_{40}$	99352$_{70}$	98574$_{94}$	97484$_{108}$	96080$_{112}$	94361$_{109}$
99863$_{34}$	99422$_{67}$	98668$_{90}$	97592$_{105}$	96192$_{111}$	94470$_{106}$
99897$_{30}$	99489$_{62}$	98758$_{87}$	97697$_{103}$	96303$_{109}$	94576$_{105}$
99927$_{24}$	99551$_{58}$	98845$_{83}$	97800$_{99}$	96412$_{106}$	94681$_{102}$
99951$_{20}$	99609$_{54}$	98928$_{81}$	97899$_{98}$	96518$_{104}$	94783$_{101}$
99971$_{15}$	99663$_{50}$	99009$_{77}$	97997$_{94}$	96622$_{102}$	94884$_{99}$
99986$_{9}$	99713$_{46}$	99086$_{73}$	98091$_{92}$	96724$_{100}$	94983$_{97}$
99995$_{5}$	99759$_{42}$	99159$_{71}$	98183$_{90}$	96824$_{97}$	95080$_{95}$
1·00000$_{1}$	99801$_{38}$	99230$_{68}$	98273$_{87}$	96921$_{96}$	95175$_{93}$
99999$_{5}$	99839$_{34}$	99298$_{64}$	98360$_{84}$	97017$_{94}$	95268$_{91}$
99994$_{11}$	99873$_{30}$	99362$_{61}$	98444$_{82}$	97111$_{92}$	95359$_{90}$
99983$_{15}$	99903$_{26}$	99423$_{58}$	98526$_{80}$	97203$_{90}$	95449$_{88}$
99968$_{21}$	99929$_{22}$	99481$_{55}$	98606$_{77}$	97293$_{88}$	95537$_{87}$
99947$_{25}$	99951$_{18}$	99536$_{52}$	98683$_{75}$	97381$_{86}$	95624$_{85}$
99922$_{31}$	99969$_{14}$	99588$_{48}$	98758$_{72}$	97467$_{84}$	95709$_{83}$
99891$_{35}$	99983$_{9}$	99636$_{46}$	98830$_{71}$	97551$_{83}$	95792$_{81}$
99856$_{41}$	99992$_{6}$	99682$_{42}$	98901$_{67}$	97634$_{80}$	95873$_{80}$
99815$_{45}$	99998$_{2}$	99724$_{40}$	98968$_{66}$	97714$_{79}$	95953$_{79}$
99770$_{51}$	1·00000$_{2}$	99764$_{36}$	99034$_{63}$	97793$_{77}$	96032$_{77}$
99719$_{56}$	99998$_{6}$	99800$_{33}$	99097$_{61}$	97870$_{76}$	96109$_{76}$
99663$_{60}$	99992$_{11}$	99833$_{30}$	99158$_{59}$	97946$_{73}$	96185$_{74}$
99603$_{66}$	99981$_{14}$	99863$_{28}$	99217$_{56}$	98019$_{72}$	96259$_{72}$
99537$_{71}$	99967$_{18}$	99891$_{24}$	99273$_{54}$	98091$_{71}$	96331$_{72}$
99466	99949	99915	99327	98162	96403
1·85407	**1·94957**	**2·07536**	**2·25721**	**2·57809**	

m	0·6		0·7		0·8		0·9		1·0	
u	sn u	Δ	sn u	Δ	sn u	Δ	sn u	Δ	sn u	Δ
2·00	99949	22	99915	21	99327	53	98162	68	96403	70
2·01	99927	26	99936	18	99380	49	98230	67	96473	68
2·02	99901	31	99954	15	99429	48	98297	66	96541	68
2·03	99870	34	99969	12	99477	46	98363	64	96609	66
2·04	99836	38	99981	9	99523	43	98427	63	96675	65
2·05	99798	42	99990	6	99566	41	98490	60	96740	63
2·06	99756	47	99996	4	99607	39	98550	60	96803	62
2·07	99709	50	1·00000		99646	38	98610	58	96865	61
2·08	99659	55	1·00000	3	99684	34	98668	56	96926	60
2·09	99604	58	99997	6	99718	33	98724	55	96986	59
2·10	99546	63	99991	9	99751	31	98779	54	97045	58
2·11	99483	67	99982	12	99782	29	98833	52	97103	56
2·12	99416	71	99970	15	99811	27	98885	51	97159	56
2·13	99345	75	99955	18	99838	24	98936	50	97215	54
2·14	99270	79	99937	21	99862	23	98986	48	97269	54
2·15	99191	83	99916	24	99885	20	99034	47	97323	52
2·16	99108	88	99892	27	99905	19	99081	45	97375	51
2·17	99020	91	99865	30	99924	16	99126	44	97426	51
2·18	98929	96	99835	33	99940	15	99170	43	97477	49
2·19	98833	100	99802	36	99955	12	99213	42	97526	48
2·20	98733	105	99766	39	99967	11	99255	40	97574	48
2·21	98628	108	99727	42	99978	8	99295	40	97622	46
2·22	98520	113	99685	45	99986	7	99335	38	97668	46
2·23	98407	118	99640	49	99993	4	99373	36	97714	45
2·24	98289	121	99591	51	99997	2	99409	36	97759	44
2·25	98168		99540		99999		99445		97803	
K	1·94957		2·07536		2·25721		2·57809			

m	0·6	0·7	0·8	0·9	1·0
u	sn *u*	sn *u*	sn *u*	sn *u*	sn *u*
2·25	98168 126	99540 55	99999 1	99445 34	97803 43
2·26	98042 130	99485 58	1·00000 2	99479 33	97846 42
2·27	97912 135	99427 61	99998 3	99512 32	97888 41
2·28	97777 139	99366 64	99995 6	99544 31	97929 41
2·29	97638 144	99302 67	99989 7	99575 30	97970 40
2·30	97494 148	99235 70	99982 10	99605 28	98010 39
2·31	97346 152	99165 74	99972 11	99633 27	98049 38
2·32	97194 158	99091 77	99961 14	99660 27	98087 37
2·33	97036 161	99014 80	99947 16	99687 25	98124 37
2·34	96875 167	98934 83	99931 17	99712 24	98161 36
2·35	96708 170	98851 87	99914 20	99736 23	98197 36
2·36	96538 176	98764 90	99894 22	99759 22	98233 34
2·37	96362 180	98674 93	99872 23	99781 21	98267 34
2·38	96182 185	98581 97	99849 26	99802 19	98301 34
2·39	95997 190	98484 99	99823 28	99821 19	98335 32
2·40	95807 194	98385 104	99795 30	99840 18	98367 33
2·41	95613 199	98281 107	99765 32	99858 16	98400 31
2·42	95414 205	98174 110	99733 34	99874 16	98431 31
2·43	95209 208	98064 114	99699 36	99890 14	98462 30
2·44	95001 214	97950 117	99663 38	99904 14	98492 30
2·45	94787 219	97833 120	99625 40	99918 12	98522 29
2·46	94568 224	97713 125	99585 43	99930 11	98551 28
2·47	94344 229	97588 127	99542 45	99941 11	98579 28
2·48	94115 234	97461 132	99497 46	99952 9	98607 28
2·49	93881 239	97329 135	99451 49	99961 8	98635 26
2·50	93642	97194	99402	99969	98661
K	1·94957	2·07536	2·25721	2·57809	

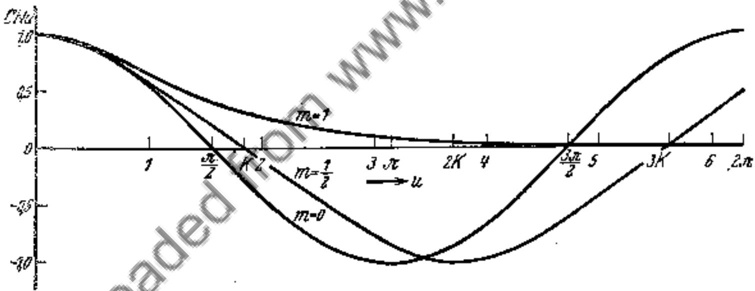
2·50 – 3·00

m / *u*	0·9 sn *u*	Δ	1·0 sn *u*	Δ
2·50	99969	8	98661	27
2·51	99977	6	98688	26
2·52	99983	5	98714	25
2·53	99988	5	98739	25
2·54	99993	3	98764	24
2·55	99996	2	98788	24
2·56	99998	2	98812	23
2·57	1·00000		98835	23
2·58	1·00000	1	98858	23
2·59	99999	1	98881	22
2·60	99998	3	98903	21
2·61	99995	4	98924	22
2·62	99991	4	98946	20
2·63	99987	6	98966	21
2·64	99981	7	98987	20
2·65	99974	8	99007	19
2·66	99966	8	99026	19
2·67	99958	10	99045	19
2·68	99948	11	99064	19
2·69	99937	12	99083	18
2·70	99925	12	99101	17
2·71	99913	14	99118	18
2·72	99899	15	99136	17
2·73	99884	16	99153	17
2·74	99868	17	99170	16
2·75	99851		99186	
K	2·57809			

m / *u*	0·9 sn *u*	Δ	1·0 sn *u*	Δ
2·75	99851	18	99186	16
2·76	99833	19	99202	16
2·77	99814	20	99218	15
2·78	99794	21	99233	15
2·79	99773	23	99248	15
2·80	99750	23	99263	15
2·81	99727	25	99278	14
2·82	99702	25	99292	14
2·83	99677	27	99306	14
2·84	99650	28	99320	13
2·85	99622	29	99333	13
2·86	99593	30	99346	13
2·87	99563	31	99359	13
2·88	99532	32	99372	12
2·89	99500	34	99384	12
2·90	99466	35	99396	12
2·91	99431	36	99408	12
2·92	99395	37	99420	11
2·93	99358	38	99431	12
2·94	99320	40	99443	11
2·95	99280	41	99454	10
2·96	99239	42	99464	11
2·97	99197	43	99475	10
2·98	99154	45	99485	11
2·99	99109	46	99496	9
3·00	99063		99505	
K	2·57809			

m	1·0		*m*	1·0
u	sn *u*		*u*	sn *u*
3·0	99505	90	5·5	99997
3·1	99595	73	5·6	99997
3·2	99668	60	5·7	99998
3·3	99728	49	5·8	99998
3·4	99777	41	5·9	99998
3·5	99818	33	6·0	99999
3·6	99851	27	6·1	99999
3·7	99878	22	6·2	99999
3·8	99900	18	6·3	99999
3·9	99918	15	6·4	99999
4·0	99933	12	6·5	1·00000
4·1	99945	10		
4·2	99955	8		
4·3	99963	7		
4·4	99970	5		
4·5	99975	5		
4·6	99980			
4·7	99983			
4·8	99986			
4·9	99989			
5·0	99991			
5·1	99993			
5·2	99994			
5·3	99995			
5·4	99996			
5·5	99997			

Five-figure Table of the Elliptic Function
cn $(u \mid m)$



m	0·0	0·1	0·2	0·3	0·4
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
0·00	1·00000$_{5}$	1·00000$_{5}$	1·00000$_{5}$	1·00000$_{5}$	1·00000$_{5}$
·01	99995$_{15}$	99995$_{15}$	99995$_{15}$	99995$_{15}$	99995$_{15}$
·02	99980$_{25}$	99980$_{25}$	99980$_{25}$	99980$_{25}$	99980$_{25}$
·03	99955$_{35}$	99955$_{35}$	99955$_{35}$	99955$_{35}$	99955$_{35}$
·04	99920$_{45}$	99920$_{45}$	99920$_{45}$	99920$_{45}$	99920$_{45}$
·05	99875$_{55}$	99875$_{55}$	99875$_{55}$	99875$_{55}$	99875$_{55}$
·06	99820$_{65}$	99820$_{65}$	99820$_{65}$	99820$_{65}$	99820$_{65}$
·07	99755$_{75}$	99755$_{75}$	99755$_{75}$	99755$_{75}$	99755$_{75}$
·08	99680$_{85}$	99680$_{85}$	99680$_{85}$	99680$_{84}$	99680$_{84}$
·09	99595$_{95}$	99595$_{94}$	99595$_{94}$	99596$_{95}$	99596$_{95}$
·10	99500$_{104}$	99501$_{105}$	99501$_{105}$	99501$_{105}$	99501$_{104}$
·11	99396$_{115}$	99396$_{115}$	99396$_{114}$	99396$_{114}$	99397$_{115}$
·12	99281$_{125}$	99281$_{124}$	99282$_{125}$	99282$_{124}$	99282$_{124}$
·13	99156$_{134}$	99157$_{135}$	99157$_{134}$	99158$_{134}$	99158$_{134}$
·14	99022$_{145}$	99022$_{144}$	99023$_{144}$	99024$_{144}$	99024$_{144}$
·15	98877$_{154}$	98878$_{154}$	98879$_{154}$	98880$_{154}$	98880$_{153}$
·16	98723$_{165}$	98724$_{164}$	98725$_{164}$	98726$_{163}$	98727$_{163}$
·17	98558$_{174}$	98560$_{174}$	98561$_{173}$	98563$_{173}$	98564$_{173}$
·18	98384$_{184}$	98386$_{183}$	98388$_{183}$	98390$_{183}$	98391$_{182}$
·19	98200$_{193}$	98203$_{194}$	98205$_{193}$	98207$_{192}$	98209$_{192}$
·20	98007$_{204}$	98009$_{203}$	98012$_{203}$	98015$_{202}$	98017$_{201}$
·21	97803$_{213}$	97806$_{212}$	97809$_{212}$	97813$_{212}$	97816$_{211}$
·22	97590$_{223}$	97594$_{223}$	97597$_{221}$	97601$_{221}$	97605$_{220}$
·23	97367$_{233}$	97371$_{232}$	97376$_{231}$	97380$_{230}$	97385$_{230}$
·24	97134$_{243}$	97139$_{241}$	97145$_{241}$	97150$_{240}$	97155$_{238}$
·25	96891	96898	96904	96910	96917
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn u	cn u	cn u	cn u	cn u	cn u
$1·00000_{5}$	$1·00000_{5}$	$1·00000_{5}$	$1·00000_{5}$	$1·00000_{5}$	$1·00000_{5}$
99995_{15}	99995_{15}	99995_{15}	99995_{15}	99995_{15}	99995_{15}
99980_{25}	99980_{25}	99980_{25}	99980_{25}	99980_{25}	99980_{25}
99955_{35}	99955_{35}	99955_{35}	99955_{35}	99955_{35}	99955_{35}
99920_{45}	99920_{45}	99920_{45}	99920_{45}	99920_{45}	99920_{45}
99875_{55}	99875_{55}	99875_{55}	99875_{55}	99875_{55}	99875_{55}
99820_{65}	99820_{65}	99820_{65}	99820_{65}	99820_{65}	99820_{65}
99755_{74}	99755_{74}	99755_{74}	99755_{74}	99755_{74}	99755_{74}
99681_{85}	99681_{85}	99681_{85}	99681_{85}	99681_{85}	99681_{85}
99596_{95}	99596_{95}	99596_{94}	99596_{94}	99596_{94}	99596_{94}
99501_{104}	99501_{104}	99502_{105}	99502_{104}	99502_{104}	99502_{104}
99397_{114}	99397_{114}	99397_{114}	99398_{114}	99398_{114}	99398_{114}
99283_{124}	99283_{124}	99283_{124}	99284_{124}	99284_{124}	99284_{123}
99159_{134}	99159_{134}	99159_{133}	99160_{133}	99160_{133}	99161_{133}
99025_{144}	99025_{143}	99026_{143}	99027_{143}	99027_{142}	99028_{143}
98881_{153}	98882_{153}	98883_{153}	98884_{153}	98885_{153}	98885_{151}
98728_{163}	98729_{162}	98730_{162}	98731_{162}	98732_{161}	98734_{162}
98565_{172}	98567_{172}	98568_{172}	98569_{171}	98571_{171}	98572_{170}
98393_{182}	98395_{182}	98396_{181}	98398_{180}	98400_{180}	98402_{180}
98211_{191}	98213_{191}	98215_{190}	98218_{190}	98220_{190}	98222_{189}
98020_{201}	98022_{200}	98025_{200}	98028_{200}	98030_{198}	98033_{198}
97819_{210}	97822_{209}	97825_{209}	97828_{208}	97832_{208}	97835_{207}
97609_{220}	97613_{219}	97616_{217}	97620_{217}	97624_{216}	97628_{216}
97389_{228}	97394_{228}	97399_{227}	97403_{226}	97408_{226}	97412_{224}
97161_{238}	97166_{237}	97172_{236}	97177_{235}	97182_{234}	97188_{234}
96923	96929	96936	96942	96948	96954
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	cn u	cn u	cn u	cn u	cn u
0·25	96891_{252}	96898_{252}	96904_{250}	96910_{249}	96917_{248}
·26	96639_{262}	96646_{260}	96654_{260}	96661_{258}	96669_{258}
·27	96377_{271}	96386_{271}	96394_{269}	96403_{268}	96411_{266}
·28	96106_{282}	96115_{279}	96125_{278}	96135_{276}	96145_{275}
·29	95824_{290}	95836_{289}	95847_{287}	95859_{286}	95870_{284}
·30	95534_{301}	95547_{299}	95560_{297}	95573_{295}	95586_{293}
·31	95233_{309}	95248_{308}	95263_{306}	95278_{304}	95293_{302}
·32	94924_{320}	94940_{317}	94957_{315}	94974_{313}	94991_{311}
·33	94604_{329}	94623_{326}	94642_{324}	94661_{322}	94680_{320}
·34	94275_{338}	94297_{336}	94318_{333}	94339_{330}	94360_{328}
·35	93937_{347}	93961_{345}	93985_{342}	94009_{340}	94032_{336}
·36	93590_{357}	93616_{354}	93643_{351}	93669_{348}	93696_{345}
·37	93233_{367}	93262_{363}	93292_{360}	93321_{356}	93351_{354}
·38	92866_{375}	92899_{372}	92932_{368}	92965_{365}	92997_{361}
·39	92491_{385}	92527_{381}	92564_{378}	92600_{374}	92636_{370}
·40	92106_{394}	92146_{390}	92186_{386}	92226_{382}	92266_{378}
·41	91712_{403}	91756_{399}	91800_{394}	91844_{390}	91888_{386}
·42	91309_{412}	91357_{407}	91406_{403}	91454_{399}	91502_{394}
·43	90897_{422}	90950_{417}	91003_{412}	91055_{406}	91108_{402}
·44	90475_{430}	90533_{425}	90591_{420}	90649_{415}	90706_{410}
·45	90045_{440}	90108_{434}	90171_{428}	90234_{423}	90296_{417}
·46	89605_{448}	89674_{442}	89743_{437}	89811_{431}	89879_{425}
·47	89157_{458}	89232_{451}	89306_{445}	89380_{439}	89454_{433}
·48	88699_{466}	88781_{460}	88861_{453}	88941_{446}	89021_{440}
·49	88233_{475}	88321_{468}	88408_{461}	88495_{454}	88581_{447}
·50	87758	87853	87947	88041	88134
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn u	cn u	cn u	cn u	cn u	cn u
96923_{247}	96929_{246}	96936_{245}	96942_{244}	96948_{243}	96954_{241}
96676_{256}	96683_{254}	96691_{254}	96698_{252}	96705_{251}	96713_{250}
96420_{265}	96429_{264}	96437_{262}	96446_{261}	96454_{260}	96463_{259}
96155_{274}	96165_{272}	96175_{271}	96185_{270}	96194_{268}	96204_{267}
95881_{283}	95893_{282}	95904_{280}	95915_{278}	95926_{276}	95937_{274}
95599_{292}	95611_{289}	95624_{288}	95637_{286}	95650_{284}	95663_{283}
95307_{300}	95322_{298}	95336_{296}	95351_{294}	95366_{293}	95380_{290}
95007_{308}	95024_{307}	95040_{304}	95057_{302}	95073_{300}	95090_{298}
94699_{318}	94717_{315}	94736_{313}	94755_{311}	94773_{308}	94792_{306}
94381_{325}	94402_{323}	94423_{320}	94444_{318}	94465_{315}	94486_{313}
94056_{334}	94079_{331}	94103_{329}	94126_{325}	94150_{323}	94173_{321}
93722_{342}	93748_{339}	93774_{336}	93801_{334}	93827_{331}	93852_{327}
93380_{350}	93409_{347}	93438_{344}	93467_{341}	93496_{338}	93525_{335}
93030_{358}	93062_{355}	93094_{351}	93126_{348}	93158_{344}	93190_{341}
92672_{367}	92707_{362}	92743_{359}	92778_{355}	92814_{352}	92849_{348}
92305_{374}	92345_{370}	92384_{366}	92423_{362}	92462_{359}	92501_{355}
91931_{382}	91975_{378}	92018_{374}	92061_{370}	92103_{365}	92146_{361}
91549_{389}	91597_{385}	91644_{380}	91691_{376}	91738_{372}	91785_{368}
91160_{397}	91212_{393}	91264_{388}	91315_{383}	91366_{378}	91417_{373}
90763_{405}	90819_{399}	90876_{395}	90932_{389}	90988_{384}	91044_{380}
90358_{412}	90420_{407}	90481_{401}	90543_{396}	90604_{391}	90664_{385}
89946_{419}	90013_{413}	90080_{408}	90147_{402}	90213_{397}	90279_{391}
89527_{426}	89600_{420}	89672_{414}	89745_{409}	89816_{402}	89888_{397}
89101_{434}	89180_{428}	89258_{421}	89336_{414}	89414_{408}	89491_{402}
88667_{440}	88752_{433}	88837_{427}	88922_{421}	89006_{414}	89089_{407}
88227	88319	88410	88501	88592	88682
1·85407	**1·94957**	**2·07536**	**2·25721**	**2·57809**	

m	0·0	0·1	0·2	0·3	0·4
u	cn u	cn u	cn u	cn u	cn u
0·50	87758_{484}	87853_{476}	87947_{469}	88041_{462}	88134_{454}
·51	87274_{492}	87377_{485}	87478_{477}	87579_{469}	87680_{462}
·52	86782_{501}	86892_{493}	87001_{485}	87110_{477}	87218_{469}
·53	86281_{510}	86399_{501}	86516_{492}	86633_{484}	86749_{475}
·54	85771_{519}	85898_{510}	86024_{500}	86149_{491}	86274_{483}
·55	85252_{526}	85388_{517}	85524_{508}	85658_{499}	85791_{489}
·56	84726_{536}	84871_{525}	85016_{516}	85159_{506}	85302_{496}
·57	84190_{544}	84346_{534}	84500_{523}	84653_{512}	84806_{503}
·58	83646_{552}	83812_{541}	83977_{530}	84141_{520}	84303_{509}
·59	83094_{560}	83271_{549}	83447_{538}	83621_{526}	83794_{515}
·60	82534_{569}	82722_{557}	82909_{545}	83095_{534}	83279_{522}
·61	81965_{577}	82165_{564}	82364_{552}	82561_{540}	82757_{528}
·62	81388_{585}	81601_{572}	81812_{559}	82021_{546}	82229_{534}
·63	80803_{593}	81029_{580}	81253_{567}	81475_{553}	81695_{540}
·64	80210_{602}	80449_{587}	80686_{573}	80922_{560}	81155_{545}
·65	79608_{609}	79862_{595}	80113_{580}	80362_{565}	80610_{552}
·66	78999_{617}	79267_{602}	79533_{587}	79797_{572}	80058_{557}
·67	78382_{625}	78665_{609}	78946_{593}	79225_{579}	79501_{563}
·68	77757_{632}	78056_{616}	78353_{601}	78646_{584}	78938_{569}
·69	77125_{641}	77440_{623}	77752_{606}	78062_{590}	78369_{574}
·70	76484_{648}	76817_{631}	77146_{613}	77472_{596}	77795_{579}
·71	75836_{655}	76186_{637}	76533_{620}	76876_{602}	77216_{585}
·72	75181_{664}	75549_{645}	75913_{626}	76274_{608}	76631_{589}
·73	74517_{670}	74904_{651}	75287_{632}	75666_{613}	76042_{595}
·74	73847_{678}	74253_{658}	74655_{638}	75053_{619}	75447_{599}
·75	73169	73595	74017	74434	74848
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn *u*	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
88227_{448}	88319_{440}	88410_{433}	88501_{426}	88592_{419}	88682_{412}
87779_{454}	87879_{447}	87977_{439}	88075_{432}	88173_{425}	88270_{418}
87325_{461}	87432_{453}	87538_{445}	87643_{437}	87748_{429}	87852_{422}
86864_{467}	86979_{459}	87093_{451}	87206_{443}	87319_{435}	87430_{426}
86397_{474}	86520_{465}	86642_{456}	86763_{448}	86884_{440}	87004_{432}
85923_{480}	86055_{471}	86186_{462}	86315_{453}	86444_{444}	86572_{435}
85443_{486}	85584_{477}	85724_{468}	85862_{458}	86000_{449}	86137_{440}
84957_{492}	85107_{482}	85256_{472}	85404_{462}	85551_{453}	85697_{443}
84465_{499}	84625_{488}	84784_{478}	84942_{468}	85098_{457}	85254_{448}
83966_{504}	84137_{494}	84306_{483}	84474_{472}	84641_{462}	84806_{451}
83462_{511}	83643_{499}	83823_{488}	84002_{477}	84179_{465}	84355_{455}
82951_{515}	83144_{504}	83335_{492}	83525_{481}	83714_{470}	83900_{458}
82436_{522}	82640_{509}	82843_{497}	83044_{485}	83244_{473}	83442_{461}
81914_{527}	82131_{514}	82346_{502}	82559_{489}	82771_{477}	82981_{465}
81387_{532}	81617_{520}	81844_{506}	82070_{493}	82294_{480}	82516_{468}
80855_{538}	81097_{523}	81338_{510}	81577_{497}	81814_{484}	82048_{470}
80317_{543}	80574_{529}	80828_{514}	81080_{500}	81330_{487}	81578_{473}
79774_{548}	80045_{533}	80314_{519}	80580_{504}	80843_{489}	81105_{476}
79226_{553}	79512_{538}	79795_{522}	80076_{508}	80354_{493}	80629_{478}
78673_{558}	78974_{542}	79273_{527}	79568_{511}	79861_{495}	80151_{480}
78115_{562}	78432_{546}	78746_{529}	79057_{513}	79366_{499}	79671_{483}
77553_{568}	77886_{550}	78217_{534}	78544_{517}	78867_{500}	79188_{485}
76985_{571}	77336_{554}	77683_{537}	78027_{520}	78367_{503}	78703_{486}
76414_{577}	76782_{558}	77146_{540}	77507_{523}	77864_{506}	78217_{488}
75837_{580}	76224_{562}	76606_{544}	76984_{525}	77358_{507}	77729_{490}
75257	75662	76062	76459	76851	77239
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
0·75	73169_{685}	73595_{665}	74017_{645}	74434_{624}	74848_{605}
·76	72484_{693}	72930_{671}	73372_{650}	73810_{630}	74243_{609}
·77	71791_{700}	72259_{678}	72722_{656}	73180_{634}	73634_{614}
·78	71091_{706}	71581_{684}	72066_{662}	72546_{640}	73020_{618}
·79	70385_{714}	70897_{691}	71404_{668}	71906_{646}	72402_{623}
·80	69671_{721}	70206_{697}	70736_{673}	71260_{650}	71779_{627}
·81	68950_{728}	69509_{703}	70063_{679}	70610_{655}	71152_{632}
·82	68222_{734}	68806_{709}	69384_{685}	69955_{660}	70520_{636}
·83	67488_{742}	68097_{716}	68699_{690}	69295_{665}	69884_{640}
·84	66746_{748}	67381_{721}	68009_{695}	68630_{669}	69244_{644}
·85	65998_{754}	66660_{728}	67314_{700}	67961_{674}	68600_{648}
·86	65244_{761}	65932_{733}	66614_{706}	67287_{679}	67952_{652}
·87	64483_{768}	65199_{739}	65908_{711}	66608_{683}	67300_{655}
·88	63715_{774}	64460_{744}	65197_{716}	65925_{687}	66645_{659}
·89	62941_{780}	63716_{751}	64481_{721}	65238_{692}	65986_{663}
·90	62161_{786}	62965_{755}	63760_{725}	64546_{696}	65323_{667}
·91	61375_{793}	62210_{762}	63035_{731}	63850_{700}	64656_{670}
·92	60582_{799}	61448_{766}	62304_{735}	63150_{704}	63986_{673}
·93	59783_{804}	60682_{772}	61569_{739}	62446_{707}	63313_{677}
·94	58979_{811}	59910_{778}	60830_{745}	61739_{712}	62636_{679}
·95	58168_{816}	59132_{782}	60085_{748}	61027_{716}	61957_{683}
·96	57352_{822}	58350_{787}	59337_{754}	60311_{719}	61274_{686}
·97	56530_{828}	57563_{793}	58583_{757}	59592_{723}	60588_{689}
·98	55702_{833}	56770_{797}	57826_{762}	58869_{727}	59899_{692}
·99	54869_{839}	55973_{802}	57064_{766}	58142_{730}	59207_{695}
1·00	54030	55171	56298	57412	58512
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn *u*	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
75257_{585}	75662_{566}	76062_{546}	76459_{528}	76851_{509}	77239_{491}
74672_{589}	75096_{569}	75516_{550}	75931_{530}	76342_{512}	76748_{493}
74083_{593}	74527_{573}	74966_{552}	75401_{533}	75830_{513}	76255_{494}
73490_{597}	73954_{576}	74414_{556}	74868_{535}	75317_{515}	75761_{495}
72893_{601}	73378_{579}	73858_{558}	74333_{537}	74802_{516}	75266_{496}
72292_{605}	72799_{583}	73300_{560}	73796_{539}	74286_{518}	74770_{497}
71687_{608}	72216_{585}	72740_{563}	73257_{541}	73768_{519}	74273_{498}
71079_{612}	71631_{589}	72177_{566}	72716_{543}	73249_{520}	73775_{498}
70467_{616}	71042_{591}	71611_{568}	72173_{544}	72729_{522}	73277_{499}
69851_{619}	70451_{594}	71043_{570}	71629_{546}	72207_{522}	72778_{499}
69232_{622}	69857_{597}	70473_{572}	71083_{548}	71685_{524}	72279_{500}
68610_{626}	69260_{600}	69901_{574}	70535_{549}	71161_{524}	71779_{500}
67984_{628}	68660_{602}	69327_{575}	69986_{550}	70637_{525}	71279_{500}
67356_{632}	68058_{604}	68752_{578}	69436_{551}	70112_{525}	70779_{500}
66724_{634}	67454_{607}	68174_{579}	68885_{552}	69587_{526}	70279_{500}
66090_{638}	66847_{609}	67595_{581}	68333_{554}	69061_{526}	69779_{499}
65452_{640}	66238_{611}	67014_{583}	67779_{554}	68535_{527}	69280_{500}
64812_{643}	65627_{614}	66431_{584}	67225_{555}	68008_{527}	68780_{499}
64169_{646}	65013_{615}	65847_{585}	66670_{556}	67481_{527}	68281_{498}
63523_{648}	64398_{617}	65262_{587}	66114_{557}	66954_{527}	67783_{498}
62875_{651}	63781_{619}	64675_{588}	65557_{557}	66427_{527}	67285_{498}
62224_{653}	63162_{621}	64087_{589}	65000_{558}	65900_{527}	66787_{497}
61571_{656}	62541_{622}	63498_{590}	64442_{558}	65373_{527}	66290_{495}
60915_{657}	61919_{624}	62908_{591}	63884_{558}	64846_{526}	65795_{496}
60258_{660}	61295_{626}	62317_{591}	63326_{559}	64320_{526}	65299_{494}
59598	60669	61726	62767	63794	64805
1·85407	**1·94957**	**2·07536**	**2·25721**	**2·57809**	

m / u	0·0 cn u	0·1 cn u	0·2 cn u	0·3 cn u	0·4 cn u
1·00	54030$_{844}$	55171$_{807}$	56298$_{770}$	57412$_{733}$	58512$_{698}$
1·01	53186$_{849}$	54364$_{811}$	55528$_{774}$	56679$_{737}$	57814$_{700}$
1·02	52337$_{855}$	53553$_{817}$	54754$_{778}$	55942$_{741}$	57114$_{703}$
1·03	51482$_{860}$	52736$_{820}$	53976$_{781}$	55201$_{743}$	56411$_{705}$
1·04	50622$_{865}$	51916$_{825}$	53195$_{786}$	54458$_{746}$	55706$_{708}$
1·05	49757$_{870}$	51091$_{830}$	52409$_{789}$	53712$_{750}$	54998$_{710}$
1·06	48887$_{875}$	50261$_{833}$	51620$_{793}$	52962$_{753}$	54288$_{713}$
1·07	48012$_{879}$	49428$_{838}$	50827$_{797}$	52209$_{755}$	53575$_{715}$
1·08	47133$_{884}$	48590$_{842}$	50030$_{800}$	51454$_{759}$	52860$_{717}$
1·09	46249$_{889}$	47748$_{846}$	49230$_{803}$	50695$_{761}$	52143$_{720}$
1·10	45360$_{894}$	46902$_{850}$	48427$_{807}$	49934$_{764}$	51423$_{721}$
1·11	44466$_{898}$	46052$_{854}$	47620$_{810}$	49170$_{767}$	50702$_{724}$
1·12	43568$_{902}$	45198$_{858}$	46810$_{814}$	48403$_{769}$	49978$_{726}$
1·13	42666$_{907}$	44340$_{861}$	45996$_{816}$	47634$_{772}$	49252$_{727}$
1·14	41759$_{910}$	43479$_{865}$	45180$_{820}$	46862$_{774}$	48525$_{730}$
1·15	40849$_{915}$	42614$_{869}$	44360$_{822}$	46088$_{777}$	47795$_{731}$
1·16	39934$_{919}$	41745$_{872}$	43538$_{826}$	45311$_{779}$	47064$_{733}$
1·17	39015$_{923}$	40873$_{876}$	42712$_{828}$	44532$_{781}$	46331$_{735}$
1·18	38092$_{926}$	39997$_{879}$	41884$_{831}$	43751$_{784}$	45596$_{736}$
1·19	37166$_{930}$	39118$_{882}$	41053$_{834}$	42967$_{786}$	44860$_{738}$
1·20	36236$_{934}$	38236$_{885}$	40219$_{837}$	42181$_{788}$	44122$_{740}$
1·21	35302$_{937}$	37351$_{888}$	39382$_{839}$	41393$_{790}$	43382$_{741}$
1·22	34365$_{941}$	36463$_{892}$	38543$_{842}$	40603$_{792}$	42641$_{743}$
1·23	33424$_{944}$	35571$_{894}$	37701$_{844}$	39811$_{794}$	41898$_{744}$
1·24	32480$_{948}$	34677$_{897}$	36857$_{847}$	39017$_{797}$	41154$_{745}$
1·25	31532	33780	36010	38220	40409
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn *u*	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
59598_{662}	60669_{627}	61726_{593}	62767_{558}	63794_{526}	64805_{493}
58936_{664}	60042_{629}	61133_{593}	62209_{559}	63268_{525}	64312_{492}
58272_{667}	59413_{629}	60540_{595}	61650_{559}	62743_{524}	63820_{490}
57605_{668}	58784_{631}	59945_{594}	61091_{559}	62219_{524}	63330_{490}
56937_{669}	58153_{633}	59351_{596}	60532_{559}	61695_{523}	62840_{488}
56268_{672}	57520_{633}	58755_{595}	59973_{559}	61172_{523}	62352_{487}
55596_{673}	56887_{635}	58160_{597}	59414_{559}	60649_{521}	61865_{485}
54923_{675}	56252_{635}	57563_{596}	58855_{558}	60128_{521}	61380_{484}
54248_{677}	55617_{637}	56967_{597}	58297_{558}	59607_{520}	60896_{482}
53571_{678}	54980_{637}	56370_{597}	57739_{557}	59087_{518}	60414_{481}
52893_{680}	54343_{638}	55773_{597}	57182_{558}	58569_{518}	59933_{479}
52213_{681}	53705_{639}	55176_{598}	56624_{556}	58051_{517}	59454_{477}
51532_{682}	53066_{640}	54578_{598}	56068_{556}	57534_{515}	58977_{475}
50850_{684}	52426_{640}	53980_{597}	55512_{556}	57019_{514}	58502_{474}
50166_{685}	51786_{641}	53383_{598}	54956_{555}	56505_{513}	58028_{471}
49481_{686}	51145_{642}	52785_{597}	54401_{554}	55992_{511}	57557_{470}
48795_{687}	50503_{642}	52188_{598}	53847_{553}	55481_{511}	57087_{468}
48108_{689}	49861_{642}	51590_{597}	53294_{553}	54970_{508}	56619_{465}
47419_{689}	49219_{643}	50993_{597}	52741_{552}	54462_{508}	56154_{464}
46730_{691}	48576_{644}	50396_{597}	52189_{551}	53954_{506}	55690_{461}
46039_{691}	47932_{644}	49799_{597}	51638_{550}	53448_{504}	55229_{460}
45348_{693}	47288_{644}	49202_{596}	51088_{549}	52944_{503}	54769_{457}
44655_{693}	46644_{644}	48606_{596}	50539_{548}	52441_{501}	54312_{455}
43962_{694}	46000_{645}	48010_{596}	49991_{548}	51940_{499}	53857_{452}
43268_{695}	45355_{645}	47414_{595}	49443_{546}	51441_{498}	53405_{451}
42573	44710	46819	48897	50943	52954
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
1·25	31532_{950}	33780_{900}	36010_{849}	38220_{797}	40409_{747}
1·26	30582_{954}	32880_{903}	35161_{851}	37423_{800}	39662_{748}
1·27	29628_{956}	31977_{905}	34310_{854}	36623_{802}	38914_{749}
1·28	28672_{960}	31072_{908}	33456_{855}	35821_{803}	38165_{751}
1·29	27712_{962}	30164_{910}	32601_{858}	35018_{805}	37414_{752}
1·30	26750_{965}	29254_{913}	31743_{860}	34213_{806}	36662_{752}
1·31	25785_{967}	28341_{915}	30883_{862}	33407_{808}	35910_{754}
1·32	24818_{970}	27426_{917}	30021_{863}	32599_{810}	35156_{755}
1·33	23848_{973}	26509_{919}	29158_{866}	31789_{811}	34401_{756}
1·34	22875_{974}	25590_{922}	28292_{867}	30978_{812}	33645_{757}
1·35	21901_{977}	24668_{923}	27425_{869}	30166_{814}	32888_{758}
1·36	20924_{979}	23745_{926}	26556_{871}	29352_{815}	32130_{759}
1·37	19945_{981}	22819_{927}	25685_{872}	28537_{816}	31371_{760}
1·38	18964_{983}	21892_{929}	24813_{874}	27721_{818}	30611_{760}
1·39	17981_{984}	20963_{930}	23939_{875}	26903_{819}	29851_{761}
1·40	16997_{987}	20033_{933}	23064_{877}	26084_{820}	29090_{762}
1·41	16010_{987}	19100_{934}	22187_{878}	25264_{821}	28328_{763}
1·42	15023_{990}	18166_{935}	21309_{880}	24443_{822}	27565_{764}
1·43	14033_{991}	17231_{937}	20429_{881}	23621_{823}	26801_{764}
1·44	13042_{992}	16294_{938}	19548_{882}	22798_{824}	26037_{765}
1·45	12050_{993}	15356_{939}	18666_{883}	21974_{825}	25272_{766}
1·46	11057_{994}	14417_{940}	17783_{884}	21149_{826}	24506_{766}
1·47	10063_{996}	13477_{942}	16899_{885}	20323_{827}	23740_{767}
1·48	09067_{996}	12535_{942}	16014_{886}	19496_{828}	22973_{767}
1·49	08071_{997}	11593_{944}	15128_{887}	18668_{828}	22206_{768}
1·50	07074	10649	14241	17840	21438
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn *u*	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
42573_{696}	44710_{645}	46819_{595}	48897_{545}	50943_{496}	52954_{448}
41877_{697}	44065_{645}	46224_{594}	48352_{544}	50447_{495}	52506_{445}
41180_{697}	43420_{645}	45630_{594}	47808_{543}	49952_{492}	52061_{444}
40483_{698}	42775_{646}	45036_{593}	47265_{541}	49460_{491}	51617_{441}
39785_{698}	42129_{645}	44443_{593}	46724_{541}	48969_{489}	51176_{438}
39087_{699}	41484_{646}	43850_{592}	46183_{539}	48480_{487}	50738_{436}
38388_{700}	40838_{645}	43258_{592}	45644_{539}	47993_{486}	50302_{434}
37688_{700}	40193_{646}	42666_{591}	45105_{537}	47507_{483}	49868_{431}
36988_{701}	39547_{645}	42075_{590}	44568_{535}	47024_{482}	49437_{428}
36287_{701}	38902_{646}	41485_{590}	44033_{535}	46542_{480}	49009_{426}
35586_{702}	38256_{645}	40895_{589}	43498_{533}	46062_{478}	48583_{423}
34884_{702}	37611_{645}	40306_{588}	42965_{531}	45584_{475}	48160_{421}
34182_{702}	36966_{645}	39718_{588}	42434_{531}	45109_{474}	47739_{418}
33480_{703}	36321_{645}	39130_{586}	41903_{529}	44635_{472}	47321_{416}
32777_{703}	35676_{644}	38544_{587}	41374_{528}	44163_{470}	46905_{413}
32074_{704}	35032_{645}	37957_{585}	40846_{526}	43693_{468}	46492_{410}
31370_{704}	34387_{644}	37372_{585}	40320_{525}	43225_{466}	46082_{408}
30666_{704}	33743_{644}	36787_{583}	39795_{524}	42759_{464}	45674_{405}
29962_{704}	33099_{644}	36204_{583}	39271_{522}	42295_{462}	45269_{402}
29258_{705}	32455_{644}	35621_{583}	38749_{521}	41833_{460}	44867_{400}
28553_{705}	31811_{643}	35038_{581}	38228_{520}	41373_{458}	44467_{397}
27848_{705}	31168_{643}	34457_{581}	37708_{518}	40915_{456}	44070_{394}
27143_{705}	30525_{643}	33876_{580}	37190_{516}	40459_{453}	43676_{391}
26438_{705}	29882_{643}	33296_{579}	36674_{516}	40006_{452}	43285_{389}
25733_{706}	29239_{642}	32717_{578}	36158_{514}	39554_{450}	42896_{386}
25027	28597	32139	35644	39104	42510
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
1·50	07074_{998}	10649_{944}	14241_{888}	17840_{829}	21438_{768}
1·51	06076_{999}	09705_{945}	13353_{889}	17011_{830}	20670_{769}
1·52	05077_{998}	08760_{946}	12464_{890}	16181_{831}	19901_{769}
1·53	04079_{1000}	07814_{946}	11574_{890}	15350_{831}	19132_{770}
1·54	03079_{1000}	06868_{947}	10684_{891}	14519_{832}	18362_{770}
1·55	02079_{999}	05921_{948}	09793_{891}	13687_{832}	17592_{770}
1·56	01080_{1000}	04973_{948}	08902_{892}	12855_{833}	16822_{771}
1·57	$+00080_{1000}$	04025_{948}	08010_{893}	12022_{833}	16051_{772}
1·58	-00920_{1000}	03077_{948}	07117_{893}	11189_{834}	15279_{771}
1·59	01920_{1000}	02129_{949}	06224_{893}	10355_{835}	14508_{772}
1·60	02920_{999}	01180_{948}	05331_{894}	09520_{834}	13736_{772}
1·61	03919_{999}	$+00232_{949}$	04437_{894}	08686_{835}	12964_{773}
1·62	04918_{999}	-00717_{949}	03543_{894}	07851_{836}	12191_{773}
1·63	05917_{998}	01666_{948}	02649_{894}	07015_{835}	11418_{773}
1·64	06915_{997}	02614_{948}	01755_{894}	06180_{836}	10645_{773}
1·65	07912_{997}	03562_{948}	$+00861_{895}$	05344_{836}	09872_{773}
1·66	08909_{995}	04510_{948}	-00034_{894}	04508_{836}	09099_{774}
1·67	09904_{995}	05458_{947}	00928_{894}	03672_{837}	08325_{774}
1·68	10899_{993}	06405_{947}	01822_{895}	02835_{836}	07551_{774}
1·69	11892_{992}	07352_{946}	02717_{894}	01999_{837}	06777_{774}
1·70	12884_{991}	08298_{945}	03611_{894}	01162_{837}	06003_{774}
1·71	13875_{990}	09243_{945}	04505_{893}	$+00325_{836}$	05229_{774}
1·72	14865_{988}	10188_{944}	05398_{894}	-00511_{837}	04455_{774}
1·73	15853_{987}	11132_{943}	06292_{892}	01348_{836}	03681_{775}
1·74	16840_{985}	12075_{942}	07184_{893}	02184_{837}	02906_{774}
1·75	17825	13017	08077	03021	02132
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn *u*	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
25027_{706}	28597_{642}	32139_{577}	35644_{512}	39104_{447}	42510_{384}
24321_{706}	27955_{642}	31562_{577}	35132_{511}	38657_{446}	42126_{381}
23615_{706}	27313_{641}	30985_{576}	34621_{510}	38211_{443}	41745_{377}
22909_{706}	26672_{641}	30409_{574}	34111_{508}	37768_{442}	41368_{376}
22203_{706}	26031_{641}	29835_{575}	33603_{507}	37326_{439}	40992_{372}
21497_{707}	25390_{640}	29260_{573}	33096_{505}	36887_{438}	40620_{370}
20790_{706}	24750_{641}	28687_{572}	32591_{504}	36449_{435}	40250_{367}
20084_{707}	24109_{639}	28115_{572}	32087_{503}	36014_{433}	39883_{365}
19377_{706}	23470_{640}	27543_{571}	31584_{501}	35581_{432}	39518_{361}
18671_{707}	22830_{639}	26972_{570}	31083_{500}	35149_{429}	39157_{359}
17964_{707}	22191_{639}	26402_{569}	30583_{498}	34720_{427}	38798_{356}
17257_{707}	21552_{639}	25833_{569}	30085_{497}	34293_{425}	38442_{354}
16550_{707}	20913_{638}	25264_{567}	29588_{496}	33868_{423}	38088_{351}
15843_{706}	20275_{638}	24697_{567}	29092_{494}	33445_{422}	37737_{348}
15137_{707}	19637_{638}	24130_{566}	28598_{493}	33023_{419}	37389_{345}
14430_{707}	18999_{637}	23564_{566}	28105_{492}	32604_{417}	37044_{343}
13723_{707}	18362_{637}	22998_{564}	27613_{490}	32187_{415}	36701_{340}
13016_{707}	17725_{637}	22434_{564}	27123_{489}	31772_{413}	36361_{337}
12309_{707}	17088_{636}	21870_{563}	26634_{488}	31359_{412}	36024_{335}
11602_{707}	16452_{637}	21307_{563}	26146_{486}	30947_{409}	35689_{332}
10895_{707}	15815_{636}	20744_{561}	25660_{486}	30538_{407}	35357_{330}
10188_{708}	15179_{635}	20183_{561}	25174_{484}	30131_{406}	35027_{326}
09480_{707}	14544_{636}	19622_{561}	24690_{482}	29725_{404}	34701_{325}
08773_{707}	13908_{635}	19061_{559}	24208_{482}	29321_{401}	34376_{321}
08066_{707}	13273_{635}	18502_{559}	23726_{480}	28920_{400}	34055_{319}
07359	12638	17943	23246	28520	33736
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·0	0·1	0·2	0·3	0·4
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
1·75	17825$_{983}$	13017$_{941}$	08077$_{892}$	03021$_{836}$	02132$_{775}$
1·76	18808$_{981}$	13958$_{940}$	08969$_{891}$	03857$_{837}$	01357$_{775}$
1·77	19789$_{979}$	14898$_{938}$	09860$_{891}$	04694$_{836}$	+ 00582$_{774}$
1·78	20768$_{977}$	15836$_{938}$	10751$_{890}$	05530$_{835}$	− 00192$_{775}$
1·79	21745$_{975}$	16774$_{936}$	11641$_{890}$	06365$_{836}$	00967$_{774}$
1·80	22720$_{973}$	17710$_{934}$	12531$_{889}$	07201$_{835}$	01741$_{775}$
1·81	23693$_{970}$	18644$_{934}$	13420$_{888}$	08036$_{835}$	02516$_{774}$
1·82	24663$_{968}$	19578$_{931}$	14308$_{887}$	08871$_{835}$	03290$_{775}$
1·83	25631$_{965}$	20509$_{930}$	15195$_{886}$	09706$_{834}$	04065$_{774}$
1·84	26596$_{963}$	21439$_{928}$	16081$_{885}$	10540$_{834}$	04839$_{774}$
1·85	27559$_{960}$	22367$_{926}$	16966$_{884}$	11374$_{833}$	05613$_{774}$
1·86	28519$_{957}$	23293$_{925}$	17850$_{883}$	12207$_{833}$	06387$_{774}$
1·87	29476$_{954}$	24218$_{922}$	18733$_{882}$	13040$_{832}$	07161$_{774}$
1·88	30430$_{951}$	25140$_{921}$	19615$_{880}$	13872$_{832}$	07935$_{774}$
1·89	31381$_{948}$	26061$_{918}$	20495$_{880}$	14704$_{831}$	08709$_{774}$
1·90	32329$_{945}$	26979$_{916}$	21375$_{878}$	15535$_{830}$	09483$_{773}$
1·91	33274$_{941}$	27895$_{914}$	22253$_{877}$	16365$_{830}$	10256$_{773}$
1·92	34215$_{938}$	28809$_{911}$	23130$_{875}$	17195$_{829}$	11029$_{773}$
1·93	35153$_{934}$	29720$_{909}$	24005$_{874}$	18024$_{828}$	11802$_{772}$
1·94	36087$_{931}$	30629$_{907}$	24879$_{872}$	18852$_{828}$	12574$_{773}$
1·95	37018$_{927}$	31536$_{904}$	25751$_{870}$	19680$_{826}$	13347$_{772}$
1·96	37945$_{923}$	32440$_{901}$	26621$_{869}$	20506$_{826}$	14119$_{772}$
1·97	38868$_{920}$	33341$_{898}$	27490$_{867}$	21332$_{825}$	14891$_{771}$
1·98	39788$_{915}$	34239$_{896}$	28357$_{866}$	22157$_{824}$	15662$_{771}$
1·99	40703$_{912}$	35135$_{893}$	29223$_{863}$	22981$_{823}$	16433$_{771}$
2·00	41615	36028	30086	23804	17204
K	1·57080	1·61244	1·65962	1·71389	1·77752

0·5	0·6	0·7	0·8	0·9	1·0
cn u	cn u	cn u	cn u	cn u	cn u
07359$_{707}$	12638$_{635}$	17943$_{559}$	23246$_{479}$	28520$_{398}$	33736$_{316}$
06652$_{707}$	12003$_{634}$	17384$_{557}$	22767$_{478}$	28122$_{396}$	33420$_{314}$
05945$_{707}$	11369$_{635}$	16827$_{558}$	22289$_{477}$	27726$_{394}$	33106$_{311}$
05238$_{707}$	10734$_{634}$	16269$_{556}$	21812$_{475}$	27332$_{393}$	32795$_{309}$
04531$_{707}$	10100$_{634}$	15713$_{556}$	21337$_{475}$	26939$_{391}$	32486$_{306}$
03824$_{707}$	09466$_{633}$	15157$_{556}$	20862$_{473}$	26548$_{388}$	32180$_{303}$
03117$_{708}$	08833$_{634}$	14601$_{555}$	20389$_{472}$	26160$_{388}$	31877$_{301}$
02409$_{707}$	08199$_{633}$	14046$_{554}$	19917$_{472}$	25772$_{385}$	31576$_{298}$
01702$_{707}$	07566$_{634}$	13492$_{554}$	19445$_{470}$	25387$_{383}$	31278$_{296}$
00995$_{707}$	06932$_{633}$	12938$_{553}$	18975$_{469}$	25004$_{382}$	30982$_{293}$
+00288$_{707}$	06299$_{633}$	12385$_{553}$	18506$_{468}$	24622$_{381}$	30689$_{291}$
−00419$_{707}$	05666$_{633}$	11832$_{553}$	18038$_{467}$	24241$_{378}$	30398$_{288}$
01126$_{707}$	05033$_{632}$	11279$_{552}$	17571$_{466}$	23863$_{377}$	30110$_{286}$
01833$_{707}$	04401$_{633}$	10727$_{551}$	17105$_{466}$	23486$_{375}$	29824$_{284}$
02540$_{707}$	03768$_{633}$	10176$_{551}$	16639$_{464}$	23111$_{374}$	29540$_{281}$
03247$_{708}$	03135$_{632}$	09625$_{551}$	16175$_{463}$	22737$_{372}$	29259$_{278}$
03955$_{707}$	02503$_{633}$	09074$_{551}$	15712$_{463}$	22365$_{370}$	28981$_{277}$
04662$_{707}$	01870$_{632}$	08523$_{550}$	15249$_{461}$	21995$_{369}$	28704$_{273}$
05369$_{707}$	01238$_{633}$	07973$_{550}$	14788$_{461}$	21626$_{367}$	28431$_{272}$
06076$_{707}$	+00605$_{632}$	07423$_{549}$	14327$_{460}$	21259$_{366}$	28159$_{269}$
06783$_{707}$	−00027$_{633}$	06874$_{550}$	13867$_{460}$	20893$_{364}$	27890$_{266}$
07490$_{707}$	00660$_{632}$	06324$_{549}$	13407$_{458}$	20529$_{363}$	27624$_{265}$
08197$_{707}$	01292$_{633}$	05775$_{549}$	12949$_{458}$	20166$_{361}$	27359$_{262}$
08904$_{707}$	01925$_{632}$	05226$_{548}$	12491$_{457}$	19805$_{360}$	27097$_{259}$
09611$_{707}$	02557$_{633}$	04678$_{549}$	12034$_{456}$	19445$_{358}$	26838$_{258}$
10318	03190	04129	11578	19087	26580
1·85407	1·94957	2·07536	2·25721	2·57809	

m	0·6	0·7	0·8	0·9	1·0
u	cn u	cn u	cn u	cn u	cn u
2·00	03190_{633}	04129_{548}	11578_{456}	19087_{357}	26580_{255}
2·01	03823_{632}	03581_{548}	11122_{455}	18730_{356}	26325_{253}
2·02	04455_{633}	03033_{548}	10667_{454}	18374_{354}	26072_{250}
2·03	05088_{633}	02485_{548}	10213_{454}	18020_{353}	25822_{249}
2·04	05721_{633}	01937_{548}	09759_{453}	17667_{352}	25573_{246}
2·05	06354_{633}	01389_{548}	09306_{453}	17315_{350}	25327_{244}
2·06	06987_{633}	00841_{547}	08853_{452}	16965_{349}	25083_{241}
2·07	07620_{634}	$+00294_{548}$	08401_{451}	16616_{348}	24842_{240}
2·08	08254_{633}	-00254_{548}	07950_{452}	16268_{346}	24602_{237}
2·09	08887_{634}	00802_{547}	07498_{450}	15922_{346}	24365_{236}
2·10	09521_{634}	01349_{548}	07048_{451}	15576_{344}	24129_{233}
2·11	10155_{634}	01897_{548}	06597_{449}	15232_{343}	23896_{231}
2·12	10789_{635}	02445_{548}	06148_{450}	14889_{342}	23665_{229}
2·13	11424_{634}	02993_{548}	05698_{449}	14547_{341}	23436_{226}
2·14	12058_{635}	03541_{549}	05249_{449}	14206_{340}	23210_{225}
2·15	12693_{635}	04090_{548}	04800_{449}	13866_{338}	22985_{223}
2·16	13328_{635}	04638_{549}	04351_{448}	13528_{338}	22762_{220}
2·17	13963_{636}	05187_{548}	03903_{448}	13190_{336}	22542_{219}
2·18	14599_{635}	05735_{549}	03455_{448}	12854_{336}	22323_{217}
2·19	15234_{636}	06284_{550}	03007_{448}	12518_{335}	22106_{214}
2·20	15870_{637}	06834_{549}	02559_{447}	12183_{333}	21892_{213}
2·21	16507_{636}	07383_{550}	02112_{448}	11850_{333}	21679_{210}
2·22	17143_{637}	07933_{550}	01664_{447}	11517_{332}	21469_{209}
2·23	17780_{637}	08483_{551}	01217_{448}	11185_{331}	21260_{207}
2·24	18417_{638}	09034_{551}	00769_{447}	10854_{330}	21053_{205}
2·25	19055	09585	00322	10524	20848
K	1·94957	2·07536	2·25721	2·57809	

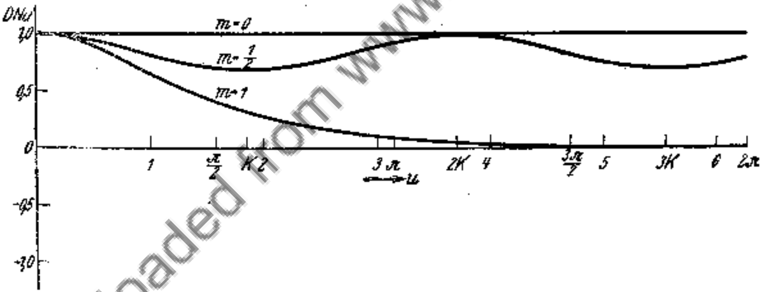
m	0·6	0·7	0·8	0·9	1·0
u	cn *u*	cn *u*	cn *u*	cn *u*	cn *u*
2·25	19053$_{639}$	09585$_{551}$	+ 00322$_{447}$	10524$_{330}$	20848$_{203}$
2·26	19692$_{638}$	10136$_{551}$	− 00125$_{447}$	10194$_{328}$	20645$_{201}$
2·27	20330$_{639}$	10687$_{552}$	00572$_{447}$	09866$_{328}$	20444$_{199}$
2·28	20969$_{638}$	11239$_{553}$	01019$_{448}$	09538$_{327}$	20245$_{197}$
2·29	21607$_{639}$	11792$_{553}$	01467$_{447}$	09211$_{326}$	20048$_{196}$
2·30	22246$_{639}$	12345$_{553}$	01914$_{448}$	08885$_{326}$	19852$_{193}$
2·31	22885$_{640}$	12898$_{554}$	02362$_{447}$	08559$_{325}$	19659$_{192}$
2·32	23525$_{640}$	13452$_{554}$	02809$_{448}$	08234$_{324}$	19467$_{190}$
2·33	24165$_{640}$	14006$_{555}$	03257$_{448}$	07910$_{324}$	19277$_{189}$
2·34	24805$_{640}$	14561$_{555}$	03705$_{448}$	07586$_{323}$	19088$_{186}$
2·35	25445$_{641}$	15116$_{556}$	04153$_{449}$	07263$_{323}$	18902$_{185}$
2·36	26086$_{641}$	15672$_{557}$	04602$_{449}$	06940$_{322}$	18717$_{183}$
2·37	26727$_{642}$	16229$_{557}$	05051$_{449}$	06618$_{321}$	18534$_{181}$
2·38	27369$_{641}$	16786$_{558}$	05500$_{449}$	06297$_{321}$	18353$_{180}$
2·39	28010$_{642}$	17344$_{558}$	05949$_{450}$	05976$_{320}$	18173$_{178}$
2·40	28652$_{643}$	17902$_{559}$	06399$_{450}$	05656$_{320}$	17995$_{176}$
2·41	29295$_{642}$	18461$_{560}$	06849$_{451}$	05336$_{320}$	17819$_{174}$
2·42	29937$_{643}$	19021$_{560}$	07300$_{451}$	05016$_{319}$	17645$_{173}$
2·43	30580$_{643}$	19581$_{561}$	07751$_{451}$	04697$_{319}$	17472$_{171}$
2·44	31223$_{644}$	20142$_{562}$	08202$_{452}$	04378$_{319}$	17301$_{170}$
2·45	31867$_{643}$	20704$_{562}$	08654$_{452}$	04059$_{318}$	17131$_{168}$
2·46	32510$_{644}$	21266$_{563}$	09106$_{453}$	03741$_{318}$	16963$_{166}$
2·47	33154$_{644}$	21829$_{564}$	09559$_{454}$	03423$_{317}$	16797$_{165}$
2·48	33798$_{645}$	22393$_{564}$	10013$_{454}$	03106$_{317}$	16632$_{163}$
2·49	34443$_{644}$	22957$_{566}$	10467$_{455}$	02789$_{317}$	16469$_{162}$
2·50	35087	23523	10922	02472	16307
K	1·94957	2·07536	2·25721	2·57809	

m	0·9	1·0	*m*	0·9	1·0
u	cn *u*	cn *u*	*u*	cn *u*	cn *u*
2·50	02472$_{317}$	16307$_{160}$	2·75	05458$_{320}$	12734$_{126}$
2·51	02155$_{317}$	16147$_{159}$	2·76	05778$_{320}$	12608$_{125}$
2·52	01838$_{317}$	15988$_{157}$	2·77	06098$_{322}$	12483$_{123}$
2·53	01521$_{316}$	15831$_{155}$	2·78	06420$_{321}$	12360$_{122}$
2·54	01205$_{317}$	15676$_{154}$	2·79	06741$_{322}$	12238$_{121}$
2·55	00888$_{316}$	15522$_{153}$	2·80	07063$_{323}$	12117$_{119}$
2·56	00572$_{316}$	15369$_{151}$	2·81	07386$_{323}$	11998$_{119}$
2·57	+00256$_{316}$	15218$_{150}$	2·82	07709$_{324}$	11879$_{117}$
2·58	−00060$_{317}$	15068$_{148}$	2·83	08033$_{325}$	11762$_{117}$
2·59	00377$_{316}$	14920$_{147}$	2·84	08358$_{325}$	11645$_{115}$
2·60	00693$_{316}$	14773$_{145}$	2·85	08683$_{326}$	11530$_{114}$
2·61	01009$_{317}$	14628$_{144}$	2·86	09009$_{327}$	11416$_{113}$
2·62	01326$_{316}$	14484$_{143}$	2·87	09336$_{327}$	11303$_{111}$
2·63	01642$_{317}$	14341$_{141}$	2·88	09663$_{328}$	11192$_{111}$
2·64	01959$_{316}$	14200$_{140}$	2·89	09991$_{329}$	11081$_{110}$
2·65	02275$_{317}$	14060$_{138}$	2·90	10320$_{330}$	10971$_{108}$
2·66	02592$_{318}$	13922$_{138}$	2·91	10650$_{330}$	10863$_{108}$
2·67	02910$_{317}$	13784$_{136}$	2·92	10980$_{331}$	10755$_{106}$
2·68	03227$_{318}$	13648$_{134}$	2·93	11311$_{333}$	10649$_{105}$
2·69	03545$_{318}$	13514$_{133}$	2·94	11644$_{333}$	10544$_{105}$
2·70	03863$_{318}$	13381$_{132}$	2·95	11977$_{334}$	10439$_{103}$
2·71	04181$_{319}$	13249$_{131}$	2·96	12311$_{335}$	10336$_{102}$
2·72	04500$_{319}$	13118$_{129}$	2·97	12646$_{336}$	10234$_{102}$
2·73	04819$_{319}$	12989$_{129}$	2·98	12982$_{337}$	10132$_{100}$
2·74	05138$_{320}$	12860$_{126}$	2·99	13319$_{338}$	10032$_{99}$
2·75	05458	12734	3·00	13657	09933
K	2·57809			2·57809	

m	1·0	m	1·0
u	cn u, dn u	u	cn u, dn u
3·00	09933_{99}	3·25	07743_{77}
3·01	09834_{97}	3·26	07666_{76}
3·02	09737_{96}	3·27	07590_{75}
3·03	09641_{96}	3·28	07515_{75}
3·04	09545_{94}	3·29	07440_{73}
3·05	09451_{94}	3·30	07367_{73}
3·06	09357_{93}	3·31	07294_{73}
3·07	09264_{92}	3·32	07221_{72}
3·08	09172_{90}	3·33	07149_{70}
3·09	09082_{90}	3·34	07079_{71}
3·10	08992_{90}	3·35	07008_{69}
3·11	08902_{88}	3·36	06939_{69}
3·12	08814_{87}	3·37	06870_{68}
3·13	08727_{87}	3·38	06802_{68}
3·14	08640_{85}	3·39	06734_{67}
3·15	08555_{85}	3·40	06667_{66}
3·16	08470_{84}	3·41	06601_{66}
3·17	08386_{83}	3·42	06535_{64}
3·18	08303_{83}	3·43	06471_{65}
3·19	08220_{81}	3·44	06406_{63}
3·20	08139_{81}	3·45	06343_{63}
3·21	08058_{80}	3·46	06280_{63}
3·22	07978_{79}	3·47	06217_{61}
3·23	07899_{78}	3·48	06156_{61}
3·24	07821_{78}	3·49	06095_{61}
3·25	07743	3·50	06034

For m = 1·0 see also pages 105, 89, 91, 93, 95, 97.

Five-figure Table of the Elliptic Function $\mathrm{dn}\,(u \mid m)$



m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
0·00	1·00000	1·00000$_{1}$	1·00000$_{1}$	1·00000$_{2}$	1·00000$_{2}$
·01	1·00000$_{2}$	99999$_{3}$	99999$_{5}$	99998$_{6}$	99998$_{8}$
·02	99998$_{2}$	99996$_{5}$	99994$_{7}$	99992$_{10}$	99990$_{12}$
·03	99996$_{4}$	99991$_{7}$	99987$_{11}$	99982$_{14}$	99978$_{18}$
·04	99992$_{4}$	99984$_{9}$	99976$_{13}$	99968$_{18}$	99960$_{22}$
·05	99988$_{6}$	99975$_{11}$	99963$_{17}$	99950$_{22}$	99938$_{28}$
·06	99982$_{6}$	99964$_{13}$	99946$_{19}$	99928$_{26}$	99910$_{32}$
·07	99976$_{8}$	99951$_{15}$	99927$_{23}$	99902$_{30}$	99878$_{38}$
·08	99968$_{8}$	99936$_{17}$	99904$_{25}$	99872$_{34}$	99840$_{42}$
·09	99960$_{10}$	99919$_{19}$	99879$_{28}$	99838$_{37}$	99798$_{47}$
·10	99950$_{10}$	99900$_{20}$	99851$_{32}$	99801$_{42}$	99751$_{52}$
·11	99940$_{12}$	99880$_{23}$	99819$_{34}$	99759$_{45}$	99699$_{57}$
·12	99928$_{12}$	99857$_{25}$	99785$_{37}$	99714$_{50}$	99642$_{62}$
·13	99916$_{13}$	99832$_{27}$	99748$_{40}$	99664$_{53}$	99580$_{66}$
·14	99903$_{15}$	99805$_{28}$	99708$_{43}$	99611$_{57}$	99514$_{72}$
·15	99888$_{15}$	99777$_{31}$	99665$_{45}$	99554$_{61}$	99442$_{76}$
·16	99873$_{16}$	99746$_{32}$	99620$_{49}$	99493$_{65}$	99366$_{81}$
·17	99857$_{17}$	99714$_{34}$	99571$_{51}$	99428$_{68}$	99285$_{85}$
·18	99840$_{18}$	99680$_{36}$	99520$_{55}$	99360$_{73}$	99200$_{90}$
·19	99822$_{19}$	99644$_{38}$	99465$_{56}$	99287$_{75}$	99110$_{95}$
·20	99803$_{20}$	99606$_{40}$	99409$_{60}$	99212$_{80}$	99015$_{99}$
·21	99783$_{21}$	99566$_{42}$	99349$_{63}$	99132$_{83}$	98916$_{104}$
·22	99762$_{22}$	99524$_{43}$	99286$_{65}$	99049$_{87}$	98812$_{109}$
·23	99740$_{22}$	99481$_{46}$	99221$_{67}$	98962$_{90}$	98703$_{112}$
·24	99718$_{24}$	99435$_{46}$	99154$_{71}$	98872$_{94}$	98591$_{118}$
·25	99694	99389	99083	98778	98473
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0
dn u	dn u	dn u	dn u	dn u
$1{\cdot}00000_{3}$	$1{\cdot}00000_{3}$	$1{\cdot}00000_{4}$	$1{\cdot}00000_{4}$	$1{\cdot}00000_{5}$
99997_{9}	99997_{11}	99996_{12}	99996_{14}	99995_{15}
99988_{15}	99986_{17}	99984_{20}	99982_{22}	99980_{25}
99973_{21}	99969_{25}	99964_{28}	99960_{32}	99955_{35}
99952_{27}	99944_{31}	99936_{36}	99928_{40}	99920_{45}
99925_{33}	99913_{39}	99900_{44}	99888_{50}	99875_{55}
99892_{39}	99874_{45}	99856_{52}	99838_{58}	99820_{65}
99853_{45}	99829_{52}	99804_{59}	99780_{67}	99755_{74}
99808_{50}	99777_{60}	99745_{68}	99713_{76}	99681_{85}
99758_{57}	99717_{66}	99677_{75}	99637_{85}	99596_{94}
99701_{62}	99651_{72}	99602_{84}	99552_{94}	99502_{104}
99639_{69}	99579_{80}	99518_{91}	99458_{102}	99398_{114}
99570_{74}	99499_{87}	99427_{98}	99356_{111}	99284_{123}
99496_{80}	99412_{93}	99329_{107}	99245_{120}	99161_{133}
99416_{85}	99319_{100}	99222_{114}	99125_{128}	99028_{143}
99331_{92}	99219_{106}	99108_{122}	98997_{137}	98885_{151}
99239_{96}	99113_{113}	98986_{129}	98860_{145}	98734_{162}
99143_{103}	99000_{120}	98857_{136}	98715_{154}	98572_{170}
99040_{108}	98880_{126}	98721_{144}	98561_{162}	98402_{180}
98932_{114}	98754_{132}	98577_{152}	98399_{170}	98222_{189}
98818_{119}	98622_{139}	98425_{158}	98229_{178}	98033_{198}
98699_{124}	98483_{145}	98267_{166}	98051_{187}	97835_{207}
98575_{130}	98338_{152}	98101_{173}	97864_{194}	97628_{216}
98445_{136}	98186_{157}	97928_{180}	97670_{202}	97412_{224}
98309_{140}	98029_{164}	97748_{187}	97468_{210}	97188_{234}
98169	97865	97561	97258	96954
1·94957	**2·07536**	**2·25721**	**2·57809**	

m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
0·50	98852_{42}	97708_{84}	96568_{126}	95431_{168}	94297_{209}
·51	98810_{42}	97624_{85}	96442_{128}	95263_{169}	94088_{211}
·52	98768_{44}	97539_{86}	96314_{129}	95094_{172}	93877_{214}
·53	98724_{43}	97453_{88}	96185_{130}	94922_{174}	93663_{217}
·54	98681_{45}	97365_{88}	96055_{132}	94748_{175}	93446_{218}
·55	98636_{44}	97277_{89}	95923_{134}	94573_{177}	93228_{221}
·56	98592_{45}	97188_{90}	95789_{134}	94396_{179}	93007_{223}
·57	98547_{46}	97098_{91}	95655_{136}	94217_{181}	92784_{225}
·58	98501_{46}	97007_{91}	95519_{137}	94036_{182}	92559_{227}
·59	98455_{46}	96916_{93}	95382_{138}	93854_{184}	92332_{229}
·60	98409_{47}	96823_{93}	95244_{139}	93670_{185}	92103_{231}
·61	98362_{47}	96730_{93}	95105_{141}	93485_{186}	91872_{232}
·62	98315_{47}	96637_{95}	94964_{141}	93299_{188}	91640_{235}
·63	98268_{48}	96542_{95}	94823_{142}	93111_{189}	91405_{235}
·64	98220_{48}	96447_{95}	94681_{143}	92922_{190}	91170_{237}
·65	98172_{48}	96352_{97}	94538_{144}	92732_{192}	90933_{239}
·66	98124_{48}	96255_{96}	94394_{145}	92540_{192}	90694_{240}
·67	98076_{49}	96159_{97}	94249_{145}	92348_{193}	90454_{241}
·68	98027_{49}	96062_{98}	94104_{146}	92155_{195}	90213_{242}
·69	97978_{49}	95964_{98}	93958_{147}	91960_{195}	89971_{244}
·70	97929_{49}	95866_{98}	93811_{147}	91765_{196}	89727_{244}
·71	97880_{50}	95768_{99}	93664_{148}	91569_{196}	89483_{245}
·72	97830_{49}	95669_{99}	93516_{148}	91373_{198}	89238_{246}
·73	97781_{50}	95570_{99}	93368_{148}	91175_{197}	88992_{247}
·74	97731_{50}	95471_{100}	93220_{149}	90978_{199}	88745_{248}
·75	97681	95371	93071	90779	88497
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0	1·0
dn u	dn u	dn u	dn u	dn u	dn 10 u, cn 10 u
93167_{250}	92041_{291}	90917_{331}	89798_{372}	88682_{412}	01348_{129}
92917_{253}	91750_{295}	90586_{336}	89426_{377}	88270_{418}	01219_{116}
92664_{256}	91455_{298}	90250_{339}	89049_{381}	87852_{422}	01103_{105}
92408_{259}	91157_{301}	89911_{344}	88668_{385}	87430_{426}	00998_{95}
92149_{262}	90856_{304}	89567_{347}	88283_{389}	87004_{432}	00903_{86}
91887_{264}	90552_{308}	89220_{350}	87894_{393}	86572_{435}	00817_{77}
91623_{267}	90244_{310}	88870_{354}	87501_{397}	86137_{440}	00740_{71}
91356_{269}	89934_{314}	88516_{357}	87104_{400}	85697_{443}	00669_{63}
91087_{272}	89620_{316}	88159_{360}	86704_{404}	85254_{448}	00606_{58}
90815_{274}	89304_{318}	87799_{363}	86300_{408}	84806_{451}	00548_{52}
90541_{276}	88986_{322}	87436_{366}	85892_{410}	84355_{455}	00496_{47}
90265_{278}	88664_{323}	87070_{369}	85482_{414}	83900_{458}	00449_{43}
89987_{280}	88341_{326}	86701_{371}	85068_{416}	83442_{461}	00406_{39}
89707_{282}	88015_{328}	86330_{374}	84652_{420}	82981_{465}	00367_{35}
89425_{284}	87687_{331}	85956_{376}	84232_{422}	82516_{468}	00332_{31}
89141_{286}	87356_{332}	85580_{379}	83810_{424}	82048_{470}	00301_{29}
88855_{287}	87024_{334}	85201_{381}	83386_{428}	81578_{473}	00272_{26}
88568_{288}	86690_{336}	84820_{382}	82958_{429}	81105_{476}	00246_{23}
88280_{290}	86354_{337}	84438_{385}	82529_{431}	80629_{478}	00223_{21}
87990_{292}	86017_{339}	84053_{387}	82098_{434}	80151_{480}	00202_{20}
87698_{292}	85678_{341}	83666_{388}	81664_{436}	79671_{483}	00182_{17}
87406_{294}	85337_{341}	83278_{389}	81228_{437}	79188_{485}	00165_{16}
87112_{295}	84996_{343}	82889_{391}	80791_{439}	78703_{486}	00149_{14}
86817_{295}	84653_{345}	82498_{393}	80352_{440}	78217_{488}	00135_{13}
86522_{297}	84308_{345}	82105_{394}	79912_{442}	77729_{490}	00122_{11}
86225	83963	81711	79470	77239	00111
1·94957	**2·07536**	**2·25721**	**2·57809**		

m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
0·75	97681_{50}	95371_{99}	93071_{150}	90779_{199}	88497_{248}
·76	97631_{50}	95272_{100}	92921_{149}	90580_{199}	88249_{248}
·77	97581_{50}	95172_{100}	92772_{150}	90381_{199}	88001_{249}
·78	97531_{50}	95072_{100}	92622_{150}	90182_{200}	87752_{250}
·79	97481_{50}	94972_{100}	92472_{150}	89982_{200}	87502_{249}
·80	97431_{50}	94872_{100}	92322_{150}	89782_{200}	87253_{250}
·81	97381_{49}	94772_{100}	92172_{150}	89582_{200}	87003_{250}
·82	97332_{50}	94672_{100}	92022_{150}	89382_{199}	86753_{250}
·83	97282_{50}	94572_{100}	91872_{149}	89183_{200}	86503_{250}
·84	97232_{50}	94472_{99}	91723_{150}	88983_{200}	86253_{250}
·85	97182_{50}	94373_{100}	91573_{150}	88783_{200}	86003_{250}
·86	97132_{49}	94273_{99}	91423_{149}	88583_{199}	85753_{249}
·87	97083_{49}	94174_{99}	91274_{149}	88384_{199}	85504_{249}
·88	97034_{50}	94075_{99}	91125_{148}	88185_{198}	85255_{249}
·89	96984_{49}	93976_{98}	90977_{148}	87987_{199}	85006_{248}
·90	96935_{48}	93878_{98}	90829_{148}	87788_{197}	84758_{248}
·91	96887_{49}	93780_{98}	90681_{147}	87591_{197}	84510_{247}
·92	96838_{48}	93682_{97}	90534_{147}	87394_{196}	84263_{246}
·93	96790_{48}	93585_{97}	90387_{146}	87198_{196}	84017_{246}
·94	96742_{48}	93488_{96}	90241_{145}	87002_{195}	83771_{245}
·95	96694_{48}	93392_{96}	90096_{145}	86807_{194}	83526_{244}
·96	96646_{47}	93296_{95}	89951_{144}	86613_{193}	83282_{243}
·97	96599_{47}	93201_{95}	89807_{143}	86420_{193}	83039_{242}
·98	96552_{47}	93106_{94}	89664_{142}	86227_{191}	82797_{241}
·99	96505_{46}	93012_{93}	89522_{142}	86036_{190}	82556_{240}
1·00	96459	92919	89380	85846	82316
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0	1·0
dn *u*	dn *u*	dn *u*	dn *u*	dn *u*	dn 10 *u*, cn 10 *u*
86225_{297}	83963_{346}	81711_{394}	79470_{443}	77239_{491}	00111_{11}
85928_{298}	83617_{347}	81317_{396}	79027_{445}	76748_{493}	00100_{9}
85630_{298}	83270_{347}	80921_{397}	78582_{445}	76255_{494}	00091_{9}
85332_{299}	82923_{349}	80524_{397}	78137_{446}	75761_{495}	00082_{8}
85033_{299}	82574_{348}	80127_{398}	77691_{448}	75266_{496}	00074_{7}
84734_{300}	82226_{350}	79729_{399}	77243_{448}	74770_{497}	00067_{6}
84434_{300}	81876_{349}	79330_{399}	76795_{448}	74273_{498}	00061_{6}
84134_{300}	81527_{350}	78931_{400}	76347_{449}	73775_{498}	00055_{5}
83834_{300}	81177_{350}	78531_{399}	75898_{450}	73277_{499}	00050_{5}
83534_{300}	80827_{350}	78132_{400}	75448_{449}	72778_{499}	00045_{4}
83234_{299}	80477_{350}	77732_{400}	74999_{450}	72279_{500}	00041
82935_{300}	80127_{350}	77332_{400}	74549_{450}	71779_{500}	00037
82635_{299}	79777_{349}	76932_{400}	74099_{450}	71279_{500}	00033
82336_{299}	79428_{350}	76532_{400}	73649_{450}	70779_{500}	00030
82037_{299}	79078_{349}	76132_{399}	73199_{450}	70279_{500}	00027
81738_{298}	78729_{348}	75733_{399}	72749_{449}	69779_{499}	00025
81440_{297}	78381_{348}	75334_{398}	72300_{449}	69280_{500}	00022
81143_{297}	78033_{347}	74936_{398}	71851_{448}	68780_{499}	00020
80846_{296}	77686_{347}	74538_{397}	71403_{448}	68281_{498}	00018
80550_{295}	77339_{345}	74141_{397}	70955_{448}	67783_{498}	00017
80255_{295}	76994_{345}	73744_{395}	70507_{446}	67285_{498}	00015
79960_{293}	76649_{344}	73349_{395}	70061_{446}	66787_{497}	00014
79667_{292}	76305_{343}	72954_{394}	69615_{444}	66290_{495}	00012
79375_{291}	75962_{342}	72560_{392}	69171_{444}	65795_{496}	00011
79084_{290}	75620_{340}	72168_{392}	68727_{443}	65299_{494}	00010
78794	75280	71776	68284	64805	00009
1·94957	2·07536	2·25721	2·57809		

m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
1·00	96459_{46}	92919_{93}	89380_{140}	85846_{190}	82316_{239}
1·01	96413_{45}	92826_{92}	89240_{140}	85656_{188}	82077_{237}
1·02	96368_{45}	92734_{91}	89100_{138}	85468_{187}	81840_{236}
1·03	96323_{45}	92643_{91}	88962_{138}	85281_{185}	81604_{235}
1·04	96278_{44}	92552_{89}	88824_{136}	85096_{185}	81369_{233}
1·05	96234_{43}	92463_{89}	88688_{136}	84911_{183}	81136_{232}
1·06	96191_{44}	92374_{88}	88552_{134}	84728_{182}	80904_{230}
1·07	96147_{42}	92286_{87}	88418_{133}	84546_{180}	80674_{229}
1·08	96105_{43}	92199_{86}	88285_{132}	84366_{178}	80445_{227}
1·09	96062_{41}	92113_{86}	88153_{130}	84188_{178}	80218_{225}
1·10	96021_{41}	92027_{84}	88023_{129}	84010_{175}	79993_{224}
1·11	95980_{41}	91943_{83}	87894_{128}	83835_{174}	79769_{222}
1·12	95939_{40}	91860_{82}	87766_{126}	83661_{173}	79547_{219}
1·13	95899_{40}	91778_{82}	87640_{125}	83488_{170}	79328_{218}
1·14	95859_{38}	91696_{80}	87515_{124}	83318_{169}	79110_{216}
1·15	95821_{39}	91616_{79}	87391_{122}	83149_{167}	78894_{214}
1·16	95782_{37}	91537_{77}	87269_{120}	82982_{165}	78680_{212}
1·17	95745_{37}	91460_{77}	87149_{119}	82817_{163}	78468_{210}
1·18	95708_{37}	91383_{75}	87030_{117}	82654_{162}	78258_{208}
1·19	95671_{35}	91308_{75}	86913_{116}	82492_{159}	78050_{205}
1·20	95636_{35}	91233_{73}	86797_{114}	82333_{158}	77845_{203}
1·21	95601_{35}	91160_{72}	86683_{112}	82175_{155}	77642_{201}
1·22	95566_{33}	91088_{70}	86571_{110}	82020_{153}	77441_{199}
1·23	95533_{33}	91018_{69}	86461_{109}	81867_{151}	77242_{196}
1·24	95500_{32}	90949_{68}	86352_{107}	81716_{149}	77046_{194}
1·25	95468	90881	86245	81567	76852
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0	1·0
dn *u*	dn *u*	dn *u*	dn *u*	dn *u*	dn 10 *u*, cn 10 *u*
78794_{289}	75280_{340}	71776_{390}	68284_{442}	64805_{493}	00009
78505_{288}	74940_{338}	71386_{389}	67842_{440}	64312_{492}	00008
78217_{286}	74602_{336}	70997_{388}	67402_{439}	63820_{490}	00007
77931_{284}	74266_{335}	70609_{386}	66963_{438}	63330_{490}	00007
77647_{283}	73931_{334}	70223_{385}	66525_{436}	62840_{488}	00006
77364_{282}	73597_{332}	69838_{383}	66089_{435}	62352_{487}	00006
77082_{280}	73265_{330}	69455_{381}	65654_{433}	61865_{485}	00005
76802_{278}	72935_{329}	69074_{380}	65221_{431}	61380_{484}	00005
76524_{276}	72606_{327}	68694_{378}	64790_{430}	60896_{482}	00004
76248_{275}	72279_{325}	68316_{376}	64360_{428}	60414_{481}	00004
75973_{273}	71954_{323}	67940_{375}	63932_{427}	59933_{479}	00003
75700_{271}	71631_{321}	67565_{372}	63505_{424}	59454_{477}	00003
75429_{268}	71310_{319}	67193_{371}	63081_{423}	58977_{475}	00003
75161_{267}	70991_{317}	66822_{368}	62658_{420}	58502_{474}	00002
74894_{265}	70674_{315}	66454_{366}	62238_{419}	58028_{471}	00002
74629_{263}	70359_{313}	66088_{364}	61819_{416}	57557_{470}	00002
74366_{260}	70046_{310}	65724_{362}	61403_{415}	57087_{468}	00002
74106_{258}	69736_{308}	65362_{360}	60988_{412}	56619_{465}	00002
73848_{256}	69428_{306}	65002_{357}	60576_{410}	56154_{464}	00002
73592_{254}	69122_{304}	64645_{355}	60166_{408}	55690_{461}	00001
73338_{251}	68818_{301}	64290_{353}	59758_{405}	55229_{460}	00001
73087_{249}	68517_{298}	63937_{350}	59353_{403}	54769_{457}	00001
72838_{246}	68219_{296}	63587_{347}	58950_{401}	54312_{455}	00001
72592_{244}	67923_{294}	63240_{345}	58549_{398}	53857_{452}	00001
72348_{241}	67629_{291}	62895_{343}	58151_{396}	53405_{451}	00001
72107	67338	62552	57755	52954	00001
$\bar{1}$·94957	2·07536	2·25721	2·57809		

m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
1·25	95468_{32}	90881_{67}	86245_{105}	81567_{147}	76852_{191}
1·26	95436_{30}	90814_{65}	86140_{103}	81420_{144}	76661_{189}
1·27	95406_{30}	90749_{63}	86037_{101}	81276_{143}	76472_{187}
1·28	95376_{29}	90686_{63}	85936_{100}	81133_{140}	76285_{184}
1·29	95347_{29}	90623_{61}	85836_{97}	80993_{137}	76101_{181}
1·30	95318_{27}	90562_{59}	85739_{96}	80856_{135}	75920_{178}
1·31	95291_{27}	90503_{58}	85643_{93}	80721_{133}	75742_{176}
1·32	95264_{26}	90445_{57}	85550_{91}	80588_{131}	75566_{173}
1·33	95238_{25}	90388_{55}	85459_{90}	80457_{128}	75393_{171}
1·34	95213_{24}	90333_{53}	85369_{87}	80329_{125}	75222_{167}
1·35	95189_{24}	90280_{52}	85282_{85}	80204_{123}	75055_{165}
1·36	95165_{23}	90228_{51}	85197_{83}	80081_{121}	74890_{162}
1·37	95142_{21}	90177_{49}	85114_{81}	79960_{118}	74728_{160}
1·38	95121_{21}	90128_{47}	85033_{79}	79842_{115}	74568_{156}
1·39	95100_{20}	90081_{46}	84954_{77}	79727_{112}	74412_{153}
1·40	95080_{20}	90035_{44}	84877_{74}	79615_{110}	74259_{151}
1·41	95060_{18}	89991_{42}	84803_{73}	79505_{108}	74108_{147}
1·42	95042_{17}	89949_{41}	84730_{70}	79397_{104}	73961_{145}
1·43	95025_{17}	89908_{39}	84660_{67}	79293_{102}	73816_{141}
1·44	95008_{15}	89869_{38}	84593_{66}	79191_{99}	73675_{138}
1·45	94993_{15}	89831_{35}	84527_{63}	79092_{97}	73537_{136}
1·46	94978_{14}	89796_{35}	84464_{61}	78995_{94}	73401_{132}
1·47	94964_{13}	89761_{32}	84403_{58}	78901_{90}	73269_{129}
1·48	94951_{12}	89729_{31}	84345_{56}	78811_{88}	73140_{126}
1·49	94939_{11}	89698_{29}	84289_{54}	78723_{86}	73014_{122}
1·50	94928	89669	84235	78637	72892
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn (u, 0) = 1

0·6 dn u	0·7 dn u	0·8 dn u	0·9 dn u	1·0 dn u	1·0 dn $10u$, cn $10u$
72107_{239}	67338_{288}	62552_{340}	57755_{393}	52954_{448}	00001
71868_{236}	67050_{286}	62212_{337}	57362_{391}	52506_{445}	00001
71632_{233}	66764_{283}	61875_{335}	56971_{388}	52061_{444}	00001
71399_{231}	66481_{280}	61540_{332}	56583_{386}	51617_{441}	00001
71168_{228}	66201_{277}	61208_{329}	56197_{382}	51176_{438}	00000
70940_{225}	65924_{274}	60879_{326}	55815_{381}	50738_{436}	
70715_{222}	65650_{272}	60553_{323}	55434_{377}	50302_{434}	
70493_{219}	65378_{269}	60230_{321}	55057_{375}	49868_{431}	
70274_{217}	65109_{265}	59909_{318}	54682_{372}	49437_{428}	
70057_{213}	64844_{263}	59591_{315}	54310_{370}	49009_{426}	
69844_{211}	64581_{260}	59276_{311}	53940_{366}	48583_{423}	
69633_{208}	64321_{256}	58965_{309}	53574_{364}	48160_{421}	
69425_{204}	64065_{254}	58656_{306}	53210_{361}	47739_{418}	
69221_{202}	63811_{251}	58350_{303}	52849_{358}	47321_{416}	
69019_{198}	63560_{247}	58047_{300}	52491_{355}	46905_{413}	
68821_{195}	63313_{244}	57747_{297}	52136_{352}	46492_{410}	
68626_{193}	63069_{241}	57450_{293}	51784_{350}	46082_{408}	
68433_{188}	62828_{238}	57157_{291}	51434_{346}	45674_{405}	
68245_{186}	62590_{235}	56866_{287}	51088_{344}	45269_{402}	
68059_{183}	62355_{231}	56579_{284}	50744_{340}	44867_{400}	
67876_{179}	62124_{228}	56295_{281}	50404_{338}	44467_{397}	
67697_{176}	61896_{225}	56014_{278}	50066_{334}	44070_{394}	
67521_{173}	61671_{221}	55736_{275}	49732_{332}	43676_{391}	
67348_{169}	61450_{218}	55461_{271}	49400_{328}	43285_{389}	
67179_{167}	61232_{215}	55190_{268}	49072_{325}	42896_{386}	
67012	61017	54922	48747	42510	
1·94957	2·07536	2·25721	2·57809		

m	0·1	0·2	0·3	0·4	0·5
u	dn u	dn u	dn u	dn u	dn u
1·50	94928_{10}	89669_{27}	84235_{52}	78637_{82}	72892_{120}
1·51	94918_{9}	89642_{26}	84183_{49}	78555_{79}	72772_{116}
1·52	94909_{8}	89616_{24}	84134_{47}	78476_{77}	72656_{113}
1·53	94901_{8}	89592_{22}	84087_{44}	78399_{74}	72543_{110}
1·54	94893_{6}	89570_{20}	84043_{42}	78325_{70}	72433_{107}
1·55	94887_{6}	89550_{19}	84001_{39}	78255_{68}	72326_{103}
1·56	94881_{4}	89531_{17}	83962_{37}	78187_{65}	72223_{100}
1·57	94877_{4}	89514_{15}	83925_{35}	78122_{62}	72123_{97}
1·58	94873_{2}	89499_{13}	83890_{32}	78060_{59}	72026_{93}
1·59	94871_{2}	89486_{12}	83858_{30}	78001_{56}	71933_{90}
1·60	94869_{1}	89474_{9}	83828_{27}	77945_{53}	71843_{87}
1·61	94868_{1}	89465_{8}	83801_{25}	77892_{50}	71756_{83}
1·62	94869_{1}	89457_{6}	83776_{22}	77842_{46}	71673_{80}
1·63	94870_{2}	89451_{5}	83754_{20}	77796_{44}	71593_{77}
1·64	94872_{3}	89446_{2}	83734_{17}	77752_{41}	71516_{73}
1·65	94875_{4}	89444_{1}	83717_{15}	77711_{38}	71443_{70}
1·66	94879_{5}	89443_{1}	83702_{12}	77673_{35}	71373_{66}
1·67	94884_{6}	89444_{2}	83690_{10}	77638_{31}	71307_{63}
1·68	94890_{7}	89446_{5}	83680_{7}	77607_{29}	71244_{59}
1·69	94897_{8}	89451_{6}	83673_{5}	77578_{25}	71185_{56}
1·70	94905_{8}	89457_{8}	83668_{2}	77553_{23}	71129_{52}
1·71	94913_{10}	89465_{10}	83666_{0}	77530_{19}	71077_{49}
1·72	94923_{11}	89475_{12}	83666_{3}	77511_{16}	71028_{46}
1·73	94934_{11}	89487_{13}	83669_{6}	77495_{14}	70982_{42}
1·74	94945_{13}	89500_{16}	83675_{7}	77481_{10}	70940_{38}
1·75	94958	89516	83682	77471	70902
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0
dn u	dn u	dn u	dn u	dn u
67012_{162}	61017_{212}	54922_{265}	48747_{323}	42510_{384}
66850_{160}	60805_{208}	54657_{261}	48424_{319}	42126_{381}
66690_{156}	60597_{204}	54396_{258}	48105_{316}	41745_{377}
66534_{152}	60393_{201}	54138_{255}	47789_{314}	41368_{376}
66382_{149}	60192_{198}	53883_{252}	47475_{310}	40992_{372}
66233_{146}	59994_{194}	53631_{248}	47165_{307}	40620_{370}
66087_{142}	59800_{190}	53383_{245}	46858_{304}	40250_{367}
65945_{139}	59610_{188}	53138_{241}	46554_{301}	39883_{365}
65806_{135}	59422_{183}	52897_{238}	46253_{297}	39518_{361}
65671_{131}	59239_{180}	52659_{235}	45956_{295}	39157_{359}
65540_{128}	59059_{177}	52424_{231}	45661_{291}	38798_{356}
65412_{125}	58882_{172}	52193_{228}	45370_{289}	38442_{354}
65287_{121}	58710_{170}	51965_{224}	45081_{285}	38088_{351}
65166_{117}	58540_{166}	51741_{221}	44796_{282}	37737_{348}
65049_{114}	58374_{162}	51520_{218}	44514_{279}	37389_{345}
64935_{110}	58212_{158}	51302_{214}	44235_{276}	37044_{343}
64825_{106}	58054_{155}	51088_{211}	43959_{273}	36701_{340}
64719_{103}	57899_{151}	50877_{207}	43686_{269}	36361_{337}
64616_{99}	57748_{148}	50670_{203}	43417_{267}	36024_{335}
64517_{96}	57600_{144}	50467_{200}	43150_{263}	35689_{332}
64421_{92}	57456_{140}	50267_{197}	42887_{260}	35357_{330}
64329_{88}	57316_{136}	50070_{193}	42627_{257}	35027_{326}
64241_{84}	57180_{133}	49877_{190}	42370_{254}	34701_{325}
64157_{81}	57047_{129}	49687_{186}	42116_{251}	34376_{321}
64076_{77}	56918_{126}	49501_{183}	41865_{247}	34055_{319}
63999	56792	49318	41618	33736
1·94957	**2·07536**	**2·25721**	**2·57809**	

m / u	0·1 dn u	0·2 dn u	0·3 dn u	0·4 dn u	0·5 dn u
1·75	94958_{13}	89516_{17}	83682_{11}	77471_{7}	70902_{35}
1·76	94971_{14}	89533_{18}	83693_{12}	77464_{3}	70867_{31}
1·77	94985_{15}	89551_{21}	83705_{16}	77461_{1}	70836_{28}
1·78	95000_{17}	89572_{22}	83721_{18}	77460_{2}	70808_{25}
1·79	95017_{16}	89594_{24}	83739_{20}	77462_{5}	70783_{21}
1·80	95033_{18}	89618_{26}	83759_{23}	77467_{9}	70762_{17}
1·81	95051_{19}	89644_{27}	83782_{25}	77476_{12}	70745_{14}
1·82	95070_{20}	89671_{29}	83807_{28}	77488_{14}	70731_{10}
1·83	95090_{20}	89700_{31}	83835_{30}	77502_{18}	70721_{7}
1·84	95110_{22}	89731_{33}	83865_{33}	77520_{21}	70714_{3}
1·85	95132_{22}	89764_{34}	83898_{35}	77541_{24}	70711
1·86	95154_{23}	89798_{36}	83933_{37}	77565_{27}	70711_{4}
1·87	95177_{24}	89834_{38}	83970_{40}	77592_{30}	70715_{8}
1·88	95201_{25}	89872_{39}	84010_{43}	77622_{33}	70723_{10}
1·89	95226_{25}	89911_{41}	84053_{45}	77655_{36}	70733_{15}
1·90	95251_{27}	89952_{43}	84098_{47}	77691_{40}	70748_{18}
1·91	95278_{27}	89995_{44}	84145_{49}	77731_{42}	70766_{21}
1·92	95305_{28}	90039_{46}	84194_{52}	77773_{45}	70787_{26}
1·93	95333_{28}	90085_{47}	84246_{55}	77818_{49}	70813_{28}
1·94	95361_{30}	90132_{49}	84301_{57}	77867_{51}	70841_{32}
1·95	95391_{30}	90181_{51}	84358_{59}	77918_{55}	70873_{36}
1·96	95421_{31}	90232_{52}	84417_{61}	77973_{57}	70909_{39}
1·97	95452_{32}	90284_{53}	84478_{64}	78030_{60}	70948_{42}
1·98	95484_{33}	90337_{55}	84542_{66}	78090_{64}	70990_{47}
1·99	95517_{33}	90392_{57}	84608_{68}	78154_{66}	71037_{49}
2·00	95550	90449	84676	78220	71086
K	1·61244	1·65962	1·71389	1·77752	1·85407

dn $(u, 0) = 1$

0·6	0·7	0·8	0·9	1·0
dn *u*	dn *u*	dn *u*	dn *u*	dn *u*
63999_{74}	56792_{121}	49318_{179}	41618_{245}	33736_{316}
63925_{69}	56671_{118}	49139_{175}	41373_{241}	33420_{314}
63856_{66}	56553_{115}	48964_{172}	41132_{238}	33106_{311}
63790_{62}	56438_{110}	48792_{169}	40894_{235}	32795_{309}
63728_{59}	56328_{107}	48623_{165}	40659_{232}	32486_{306}
63669_{54}	56221_{103}	48458_{161}	40427_{229}	32180_{303}
63615_{51}	56118_{99}	48297_{158}	40198_{226}	31877_{301}
63564_{48}	56019_{96}	48139_{155}	39972_{222}	31576_{298}
63516_{43}	55923_{91}	47984_{150}	39750_{220}	31278_{296}
63473_{40}	55832_{88}	47834_{148}	39530_{216}	30982_{293}
63433_{35}	55744_{84}	47686_{143}	39314_{213}	30689_{291}
63398_{32}	55660_{81}	47543_{141}	39101_{210}	30398_{288}
63366_{29}	55579_{76}	47402_{136}	38891_{207}	30110_{286}
63337_{24}	55503_{73}	47266_{133}	38684_{204}	29824_{284}
63313_{21}	55430_{69}	47133_{130}	38480_{201}	29540_{281}
63292_{17}	55361_{65}	47003_{126}	38279_{198}	29259_{278}
63275_{13}	55296_{61}	46877_{122}	38081_{194}	28981_{277}
63262_{9}	55235_{58}	46755_{119}	37887_{192}	28704_{273}
63253_{6}	55177_{54}	46636_{115}	37695_{188}	28431_{272}
63247_{1}	55123_{50}	46521_{112}	37507_{186}	28159_{269}
63246_{2}	55073_{46}	46409_{108}	37321_{182}	27890_{266}
63248_{5}	55027_{42}	46301_{104}	37139_{179}	27624_{265}
63253_{10}	54985_{38}	46197_{101}	36960_{177}	27359_{262}
63263_{14}	54947_{35}	46096_{98}	36783_{173}	27097_{259}
63277_{17}	54912_{31}	45998_{93}	36610_{170}	26838_{258}
63294	54881	45905	36440	26580
1·94957	2·07536	2·25721	2·57809	

m	0·6	0·7	0·8	0·9	1·0
u	dn *u*	dn *u*	dn *u*	dn *u*	dn *u*
2·00	63294_{21}	54881_{27}	45905_{91}	36440_{167}	26580_{255}
2·01	63315_{25}	54854_{23}	45814_{86}	36273_{164}	26325_{253}
2·02	63340_{28}	54831_{19}	45728_{83}	36109_{161}	26072_{250}
2·03	63368_{33}	54812_{16}	45645_{80}	35948_{158}	25822_{249}
2·04	63401_{36}	54796_{11}	45565_{76}	35790_{155}	25573_{246}
2·05	63437_{40}	54785_{8}	45489_{72}	35635_{152}	25327_{244}
2·06	63477_{43}	54777_{4}	45417_{69}	35483_{149}	25083_{241}
2·07	63520_{48}	54773	45348_{65}	35334_{146}	24842_{240}
2·08	63568_{51}	54773_{3}	45283_{62}	35188_{143}	24602_{237}
2·09	63619_{55}	54776_{8}	45221_{58}	35045_{140}	24365_{236}
2·10	63674_{59}	54784_{11}	45163_{54}	34905_{137}	24129_{233}
2·11	63733_{62}	54795_{15}	45109_{51}	34768_{134}	23896_{231}
2·12	63795_{67}	54810_{19}	45058_{47}	34634_{131}	23665_{229}
2·13	63862_{70}	54829_{23}	45011_{44}	34503_{128}	23436_{226}
2·14	63932_{73}	54852_{27}	44967_{40}	34375_{125}	23210_{225}
2·15	64005_{78}	54879_{31}	44927_{37}	34250_{122}	22985_{223}
2·16	64083_{81}	54910_{34}	44890_{33}	34128_{119}	22762_{220}
2·17	64164_{85}	54944_{38}	44857_{29}	34009_{117}	22542_{219}
2·18	64249_{88}	54982_{42}	44828_{26}	33892_{113}	22323_{217}
2·19	64337_{92}	55024_{46}	44802_{22}	33779_{110}	22106_{214}
2·20	64429_{96}	55070_{49}	44780_{19}	33669_{108}	21892_{213}
2·21	64525_{100}	55119_{54}	44761_{15}	33561_{104}	21679_{210}
2·22	64625_{103}	55173_{57}	44746_{11}	33457_{101}	21469_{209}
2·23	64728_{107}	55230_{61}	44735_{8}	33356_{99}	21260_{207}
2·24	64835_{110}	55291_{65}	44727_{5}	33257_{96}	21053_{205}
2·25	64945	55356	44722	33161	20848
K	1·94957	2·07536	2·25721	2·57809	

m	0·6	0·7	0·8	0·9	1·0
u	dn *u*	dn *u*	dn *u*	dn *u*	dn *u*
2·25	64945_{114}	55356_{69}	44722	33161_{92}	20848_{203}
2·26	65059_{118}	55425_{72}	44722_{2}	33069_{90}	20645_{201}
2·27	65177_{121}	55497_{77}	44724_{7}	32979_{87}	20444_{199}
2·28	65298_{125}	55574_{80}	44731_{10}	32892_{84}	20245_{197}
2·29	65423_{128}	55654_{84}	44741_{13}	32808_{81}	20048_{196}
2·30	65551_{132}	55738_{87}	44754_{17}	32727_{78}	19852_{193}
2·31	65683_{135}	55825_{92}	44771_{21}	32649_{76}	19659_{192}
2·32	65818_{139}	55917_{95}	44792_{24}	32573_{72}	19467_{190}
2·33	65957_{143}	56012_{99}	44816_{28}	32501_{70}	19277_{189}
2·34	66100_{146}	56111_{102}	44844_{31}	32431_{66}	19088_{186}
2·35	66246_{149}	56213_{107}	44875_{35}	32365_{64}	18902_{185}
2·36	66395_{153}	56320_{110}	44910_{39}	32301_{61}	18717_{183}
2·37	66548_{156}	56430_{114}	44949_{42}	32240_{58}	18534_{181}
2·38	66704_{160}	56544_{118}	44991_{46}	32182_{55}	18353_{180}
2·39	66864_{163}	56662_{121}	45037_{49}	32127_{52}	18173_{178}
2·40	67027_{166}	56783_{125}	45086_{53}	32075_{50}	17995_{176}
2·41	67193_{170}	56908_{129}	45139_{56}	32025_{46}	17819_{174}
2·42	67363_{173}	57037_{133}	45195_{60}	31979_{44}	17645_{173}
2·43	67536_{176}	57170_{136}	45255_{64}	31935_{41}	17472_{171}
2·44	67712_{180}	57306_{140}	45319_{67}	31894_{38}	17301_{170}
2·45	67892_{183}	57446_{144}	45386_{71}	31856_{35}	17131_{168}
2·46	68075_{186}	57590_{147}	45457_{74}	31821_{32}	16963_{166}
2·47	68261_{189}	57737_{151}	45531_{78}	31789_{29}	16797_{165}
2·48	68450_{192}	57888_{154}	45609_{82}	31760_{27}	16632_{163}
2·49	68642_{196}	58042_{159}	45691_{85}	31733_{23}	16469_{162}
2·50	68838	58201	45776	31710	16307
K	1·94957	2·07536	2·25721	2·57809	

m	0·9	1·0	m	0·9	1·0
u	dn u	dn u	u	dn u	dn u
2·50	31710_{21}	16307_{160}	2·75	32044_{50}	12734_{126}
2·51	31689_{18}	16147_{159}	2·76	32094_{54}	12608_{125}
2·52	31671_{15}	15988_{157}	2·77	32148_{56}	12483_{123}
2·53	31656_{13}	15831_{155}	2·78	32204_{59}	12360_{122}
2·54	31643_{9}	15676_{154}	2·79	32263_{62}	12238_{121}
2·55	31634_{7}	15522_{153}	2·80	32325_{65}	12117_{119}
2·56	31627_{3}	15369_{151}	2·81	32390_{68}	11998_{119}
2·57	31624_{1}	15218_{150}	2·82	32458_{70}	11879_{117}
2·58	31623_{2}	15068_{148}	2·83	32528_{74}	11762_{117}
2·59	31625_{5}	14920_{147}	2·84	32602_{76}	11645_{115}
2·60	31630_{7}	14773_{145}	2·85	32678_{79}	11530_{114}
2·61	31637_{11}	14628_{144}	2·86	32757_{83}	11416_{113}
2·62	31648_{13}	14484_{143}	2·87	32840_{85}	11303_{111}
2·63	31661_{16}	14341_{141}	2·88	32925_{88}	11192_{111}
2·64	31677_{19}	14200_{140}	2·89	33013_{91}	11081_{110}
2·65	31696_{22}	14060_{138}	2·90	33104_{93}	10971_{108}
2·66	31718_{25}	13922_{138}	2·91	33197_{97}	10863_{108}
2·67	31743_{28}	13784_{136}	2·92	33294_{100}	10755_{106}
2·68	31771_{30}	13648_{134}	2·93	33394_{103}	10649_{105}
2·69	31801_{33}	13514_{133}	2·94	33497_{105}	10544_{105}
2·70	31834_{37}	13381_{132}	2·95	33602_{109}	10439_{103}
2·71	31871_{39}	13249_{131}	2·96	33711_{111}	10336_{102}
2·72	31910_{41}	13118_{129}	2·97	33822_{114}	10234_{102}
2·73	31951_{45}	12989_{129}	2·98	33936_{118}	10132_{100}
2·74	31996_{48}	12860_{126}	2·99	34054_{120}	10032_{99}
2·75	32044	12734	3·00	34174	09933
K	2·57809			2·57809	

m	1·0	*m*	1·0
u	dn *u*, cn *u*	*u*	dn *u*, cn *u*
3·50	06034_{60}	3·75	04701_{47}
3·51	05974_{59}	3·76	04654_{46}
3·52	05915_{59}	3·77	04608_{46}
3·53	05856_{58}	3·78	04562_{45}
3·54	05798_{58}	3·79	04517_{45}
3·55	05740_{57}	3·80	04472_{45}
3·56	05683_{56}	3·81	04427_{44}
3·57	05627_{56}	3·82	04383_{43}
3·58	05571_{56}	3·83	04340_{43}
3·59	05515_{54}	3·84	04297_{43}
3·60	05461_{55}	3·85	04254_{42}
3·61	05406_{53}	3·86	04212_{42}
3·62	05353_{53}	3·87	04170_{42}
3·63	05300_{53}	3·88	04128_{41}
3·64	05247_{52}	3·89	04087_{40}
3·65	05195_{52}	3·90	04047_{41}
3·66	05143_{51}	3·91	04006_{39}
3·67	05092_{51}	3·92	03967_{40}
3·68	05041_{50}	3·93	03927_{39}
3·69	04991_{49}	3·94	03888_{38}
3·70	04942_{49}	3·95	03850_{39}
3·71	04893_{49}	3·96	03811_{38}
3·72	04844_{48}	3·97	03773_{37}
3·73	04796_{48}	3·98	03736_{37}
3·74	04748_{47}	3·99	03699_{37}
3·75	04701	4·00	03662

For m = 1·0 see also pages 83, 89, 91, 93, 95, 97.

0·00 – 0·25

m	K	K'	E
0·00	1·5707963	∞	1·5707963
·01	1·5747456	3·6956374	1·5668619
·02	1·5787399	3·3541414	1·5629126
·03	1·5827803	3·1558749	1·5589482
·04	1·5868678	3·0161125	1·5549685
·05	1·5910035	2·9083372	1·5509734
·06	1·5951882	2·8207525	1·5469625
·07	1·5994232	2·7470730	1·5429357
·08	1·6037097	2·6835514	1·5388927
·09	1·6080486	2·6277733	1·5348335
·10	1·6124413	2·5780921	1·5307576
·11	1·6168891	2·5333345	1·5266650
·12	1·6213931	2·4926353	1·5225554
·13	1·6259548	2·4553380	1·5184285
·14	1·6305755	2·4209330	1·5142840
·15	1·6352567	2·3890165	1·5101218
·16	1·6399999	2·3592636	1·5059416
·17	1·6448065	2·3314086	1·5017431
·18	1·6496782	2·3052317	1·4975260
·19	1·6546167	2·2805491	1·4932901
·20	1·6596236	2·2572053	1·4890351
·21	1·6647008	2·2350678	1·4847606
·22	1·6698501	2·2140225	1·4804664
·23	1·6750734	2·1939709	1·4761521
·24	1·6803728	2·1748271	1·4718175
·25	1·6857504	2·1565156	1·4674622
m_1	K'	K	E'

E'	q	q_1	m_1
1·0000000	0·00000000	1·00000000	1·00
1·0159935	00062815	0·26219627	0·99
1·0285945	00126267	22793457	·98
1·0399469	00190369	20687981	·97
1·0505022	00255135	19149631	·96
1·0604737	00320579	17931601	·95
1·0699861	00386714	16920753	·94
1·0791214	00453554	16055420	·93
1·0879375	00521116	15298148	·92
1·0964775	00589414	14624427	·91
1·1047747	00658465	14017313	·90
1·1128556	00728285	13464588	·89
1·1207417	00798891	12957147	·88
1·1284507	00870300	12488012	·87
1·1359978	00942531	12051720	·86
1·1433958	01015604	11643906	·85
1·1506556	01089536	11261032	·84
1·1577870	01164349	10900183	·83
1·1647983	01240064	10558935	·82
1·1716971	01316702	10235242	·81
1·1784899	01394286	09927370	·80
1·1851829	01472839	09633827	·79
1·1917813	01552385	09353329	·78
1·1982901	01632949	09084754	·77
1·2047136	01714558	08827124	·76
1·2110560	01797239	08579573	·75
E	q_1	q	m

m	K	K′	E
·25	1·6857504	2·1565156	1·4674622
·26	1·6912082	2·1389702	1·4630859
·27	1·6967486	2·1221319	1·4586882
·28	1·7023740	2·1059483	1·4542687
·29	1·7080867	2·0903727	1·4498271
·30	1·7138894	2·0753631	1·4453631
·31	1·7197848	2·0608816	1·4408761
·32	1·7257756	2·0468941	1·4363659
·33	1·7318648	2·0333694	1·4318319
·34	1·7380554	2·0202794	1·4272738
·35	1·7443506	2·0075984	1·4226911
·36	1·7507538	1·9953028	1·4180834
·37	1·7572685	1·9833710	1·4134501
·38	1·7638984	1·9717832	1·4087908
·39	1·7706473	1·9605210	1·4041050
·40	1·7775194	1·9495677	1·3993921
·41	1·7845188	1·9389077	1·3946517
·42	1·7916501	1·9285263	1·3898830
·43	1·7989180	1·9184103	1·3850856
·44	1·8063276	1·9085470	1·3802588
·45	1·8138839	1·8989249	1·3754020
·46	1·8215927	1·8895331	1·3705145
·47	1·8294598	1·8803614	1·3655957
·48	1·8374914	1·8714002	1·3606448
·49	1·8456940	1·8626408	1·3556611
·50	1·8540747	1·8540747	1·3506439
m_1	K′	K	E′

E'	q	q_1	m_1
1·2110560	01797239	08579573	·75
1·2173210	01881019	08341339	·74
1·2235118	01965929	08111742	·73
1·2296318	02051998	07890173	·72
1·2356838	02139259	07676087	·71
1·2416706	02227744	07468994	·70
1·2475945	02317488	07268450	·69
1·2534581	02408527	07074051	·68
1·2592634	02500898	06885431	·67
1·2650126	02594641	06702255	·66
1·2707075	02689797	06524218	·65
1·2763499	02786408	06351039	·64
1·2819417	02884519	06182460	·63
1·2874843	02984178	06018242	·62
1·2929792	03085432	05858165	·61
1·2984280	03188335	05702026	·60
1·3038320	03292939	05549636	·59
1·3091924	03399302	05400819	·58
1·3145106	03507483	05255411	·57
1·3197876	03617546	05113261	·56
1·3250245	03729556	04974226	·55
1·3302225	03843582	04838173	·54
1·3353824	03959700	04704976	·53
1·3405054	04077985	04574520	·52
1·3455922	04198520	04446693	·51
1·3506439	04321392	04321392	·50
E	q_1	q	m

Seven-figure Table of the Jacobian Zeta-function $Z(u)$

m	0·1		0·2		0·3		0·4	
u	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −
0·00	0·0000 000	0	0·0000 000	0	0·0000 000	0	0·0000 000	0
·01	005 066	3	010 278	4	015 667	7	021 271	7
·02	010 129	3	020 552	8	031 327	11	042 535	17
·03	015 189	7	030 818	12	046 976	18	063 782	23
·04	020 242	7	041 072	16	062 607	25	085 006	33
·05	025 288	11	051 310	19	078 213	29	106 197	39
·06	030 323	11	061 529	25	093 790	36	127 349	48
·07	035 347	15	071 723	28	109 331	42	148 453	56
·08	040 356	15	081 889	31	124 830	48	169 501	64
·09	045 350	18	092 024	37	140 281	53	190 485	71
·10	050 326	20	102 122	39	155 679	60	211 398	79
·11	055 282	22	112 181	43	171 017	65	232 232	88
·12	060 216	24	122 197	48	186 290	71	252 978	94
·13	065 126	25	132 165	51	201 492	77	273 630	102
·14	070 011	28	142 082	56	216 617	82	294 180	111
·15	074 868	29	151 943	58	231 660	89	314 619	116
·16	079 696	32	161 746	63	246 614	94	334 942	126
·17	084 492	33	171 486	67	261 474	99	355 139	133
·18	089 255	35	181 159	69	276 235	106	375 203	139
·19	093 983	38	190 763	75	290 890	109	395 128	146
·20	098 673	38	200 292	76	305 436	117	414 907	156
·21	103 325	40	209 745	82	319 865	121	434 530	160
·22	107 937	44	219 116	85	334 173	126	453 993	168
·23	112 505	43	228 402	88	348 355	133	473 288	176
·24	117 030	46	237 600	92	362 404	136	492 407	182
·25	121 509	49	246 706	94	376 317	142	511 344	188
·26	125 939	48	255 718	99	390 088	148	530 093	196
·27	130 321	52	264 631	102	403 711	152	548 646	201
·28	134 651	53	273 442	105	417 182	156	566 998	209
·29	138 928	54	282 148	109	430 497	163	585 741	215
·30	143 151	56	290 745	111	443 649	166	603 069	220
·31	147 318	58	299 231	116	456 635	171	620 777	227
·32	151 427	60	307 601	117	469 450	176	638 258	233
·33	155 476	60	315 854	121	482 089	180	655 506	238
·34	159 465	63	323 986	123	494 548	184	672 516	245
·35	163 391	64	331 995	128	506 823	190	689 281	251
·36	167 253	65	339 876	130	518 908	193	705 795	253
·37	171 050	67	347 627	132	530 800	197	722 056	262
·38	174 780	68	355 246	136	542 495	201	738 055	266
·39	178 442	70	362 729	138	553 989	205	753 788	272
·40	182 034	71	370 074	140	565 278	210	769 249	275
·41	185 555	72	377 279	144	576 357	212	784 435	281
·42	189 004	74	384 340	145	587 224	217	799 340	285
·43	192 379	75	391 256	149	597 874	220	813 960	291
·44	195 679	76	398 023	151	608 304	223	828 289	294
·45	198 903	78	404 639	152	618 511	227	842 324	299
·46	202 049	78	411 103	156	628 491	230	856 060	302
·47	205 117	80	417 411	157	638 241	233	869 494	308
·48	208 105	81	423 562	160	647 758	236	882 620	310
·49	211 012	82	429 553	161	657 039	239	895 436	314
0·50	0·0213 837	82	0·0435 383	164	0·0666 081	243	0·0907 938	319

m	0·5		0·6		0·7		0·8	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''
		−		−		−		−
0·00	0·0000 000	0	0·0000 000	0	0·0000 000	0	0·0000 000	0
·01	0027 151	10	0033 397	12	0040 169	15	0047 787	16
·02	0054 292	20	0066 782	23	0080 323	27	0095 558	31
·03	0081 413	30	0100 144	37	0120 450	43	0143 298	49
·04	0108 504	40	0133 469	48	0160 534	55	0190 989	64
·05	0135 555	50	0166 746	59	0200 563	70	0238 616	79
·06	0162 556	59	0199 964	73	0240 522	84	0286 164	96
·07	0189 498	70	0233 109	82	0280 397	97	0333 616	111
·08	0216 370	80	0266 172	97	0320 175	111	0380 957	128
·09	0243 162	89	0299 138	106	0359 842	125	0428 170	141
·10	0269 865	99	0331 998	119	0399 384	139	0475 242	159
·11	0296 469	109	0364 739	130	0438 787	151	0522 155	174
·12	0322 964	117	0397 350	142	0478 039	166	0568 894	188
·13	0349 342	129	0429 819	153	0517 125	178	0615 445	204
·14	0375 591	137	0462 135	165	0556 033	192	0661 792	219
·15	0401 703	147	0494 286	175	0594 749	205	0707 920	233
·16	0427 668	155	0526 262	188	0633 260	217	0753 815	249
·17	0453 478	166	0558 050	197	0671 554	231	0799 461	262
·18	0479 122	174	0589 641	209	0709 617	243	0844 845	278
·19	0504 592	184	0621 023	219	0747 437	255	0889 951	290
·20	0529 878	192	0652 186	230	0785 002	267	0934 767	306
·21	0554 972	200	0683 119	241	0822 300	281	0979 277	318
·22	0579 866	210	0713 811	251	0859 317	291	1023 469	333
·23	0604 550	218	0744 252	260	0896 043	303	1067 328	345
·24	0629 016	227	0774 433	271	0932 466	315	1110 842	359
·25	0653 255	235	0804 343	282	0968 574	327	1153 997	371
·26	0677 259	243	0833 971	289	1004 355	337	1196 781	385
·27	0701 020	251	0863 310	301	1039 799	348	1239 180	395
·28	0724 530	259	0892 348	309	1074 895	360	1281 184	408
·29	0747 781	267	0921 077	318	1109 631	368	1322 780	421
·30	0770 765	275	0949 488	328	1143 999	381	1363 955	431
·31	0793 474	281	0977 571	335	1177 986	389	1404 699	443
·32	0815 902	289	1005 319	346	1211 584	400	1445 000	453
·33	0838 041	297	1032 721	352	1244 782	409	1484 848	465
·34	0859 883	303	1059 771	362	1277 571	418	1524 231	474
·35	0881 422	309	1086 459	369	1309 942	428	1563 140	485
·36	0902 652	318	1112 778	377	1341 885	436	1601 564	495
·37	0923 564	323	1138 720	385	1373 392	445	1639 493	504
·38	0944 153	329	1164 277	392	1404 454	453	1676 918	513
·39	0964 413	337	1189 442	398	1435 063	462	1713 830	522
·40	0984 336	340	1214 209	407	1465 210	468	1750 220	530
·41	1003 919	349	1238 569	412	1494 889	477	1786 080	540
·42	1023 153	352	1262 517	420	1524 091	483	1821 400	547
·43	1042 035	359	1286 045	425	1552 810	492	1856 173	554
·44	1060 558	363	1309 148	431	1581 037	496	1890 392	562
·45	1078 718	370	1331 820	438	1608 768	505	1924 049	569
·46	1096 508	374	1354 054	443	1635 994	510	1957 137	575
·47	1113 924	378	1375 845	448	1662 710	516	1989 650	582
·48	1130 962	383	1397 188	453	1688 910	522	2021 581	589
·49	1147 617	387	1418 078	459	1714 588	527	2052 923	593
0·50	0·1163 885	392	0·1438 509	463	0·1739 739	533	0·2083 672	600

m	0·9		1·0		m	0·1		0·2	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	u	$Z(u)$	Δ''	$Z(u)$	Δ''
		−		−			−		−
0·00	0·0000 000	0	0·0000 000	0	0·50	0·0213 837	82	0·0435 383	164
·01	0057 145	19	0099 997	21	·51	216 580	85	441 049	166
·02	0114 271	35	0199 973	39	·52	219 238	84	446 549	167
·03	0171 362	55	0299 910	60	·53	221 812	87	451 882	168
·04	0228 398	71	0399 787	80	·54	224 299	86	457 047	172
·05	0285 363	90	0499 584	100	·55	226 700	88	462 040	172
·06	0342 238	107	0599 281	119	·56	229 013	89	466 861	175
·07	0399 006	126	0698 859	139	·57	231 237	88	471 507	174
·08	0455 648	140	0798 298	159	·58	233 373	91	475 979	178
·09	0512 150	165	0897 578	178	·59	235 418	91	480 273	178
·10	0568 487	176	0996 680	197	·60	237 372	91	484 389	179
·11	0624 648	194	1095 585	217	·61	239 235	92	488 326	181
·12	0680 615	213	1194 273	235	·62	241 006	93	492 082	182
·13	0736 369	228	1292 726	254	·63	242 684	93	495 656	183
·14	0791 895	247	1390 925	274	·64	244 269	94	499 047	184
·15	0847 174	262	1488 850	290	·65	245 760	95	502 254	185
·16	0902 191	278	1586 485	309	·66	247 156	94	505 276	185
·17	0956 930	296	1683 811	328	·67	248 458	96	508 113	186
·18	1011 373	311	1780 809	345	·68	249 664	95	510 764	188
·19	1065 505	326	1877 462	362	·69	250 775	96	513 227	187
·20	1119 311	343	1973 753	379	·70	251 790	97	515 503	189
·21	1172 774	358	2069 665	396	·71	252 708	96	517 590	188
·22	1225 879	372	2165 181	413	·72	253 530	97	519 489	189
·23	1278 612	387	2260 284	429	·73	254 255	97	521 199	190
·24	1330 958	403	2354 958	445	·74	254 883	98	522 719	189
·25	1382 901	415	2449 187	461	·75	255 413	96	524 050	191
·26	1434 429	430	2542 955	475	·76	255 847	98	525 190	189
·27	1485 527	444	2636 248	490	·77	256 183	98	526 141	190
·28	1536 181	457	2729 051	506	·78	256 421	97	526 902	190
·29	1586 378	470	2821 348	519	·79	256 562	97	527 473	190
·30	1636 105	483	2913 126	533	·80	256 606	98	527 854	190
·31	1685 349	494	3004 371	547	·81	256 552	98	528 045	190
·32	1734 099	508	3095 069	559	·82	256 400	96	528 046	188
·33	1782 341	518	3185 208	573	·83	256 152	98	527 859	190
·34	1830 065	530	3274 774	586	·84	255 806	96	527 482	188
·35	1877 259	542	3363 754	594	·85	255 364	97	526 917	187
·36	1923 911	551	3452 140	609	·86	254 825	96	526 165	188
·37	1970 012	563	3539 917	619	·87	254 190	96	525 225	187
·38	2015 550	572	3627 075	631	·88	253 459	96	524 098	186
·39	2060 516	581	3713 602	639	·89	252 632	95	522 785	185
·40	2104 901	592	3799 490	651	·90	251 710	95	521 287	185
·41	2148 694	599	3884 727	660	·91	250 693	94	519 604	183
·42	2191 888	609	3969 304	668	·92	249 582	94	517 738	183
·43	2234 473	617	4053 213	678	·93	248 377	94	515 689	181
·44	2276 441	624	4136 444	685	·94	247 078	92	513 459	181
·45	2317 785	633	4218 990	694	·95	245 687	92	511 048	179
·46	2358 496	639	4300 842	701	·96	244 204	92	508 458	179
·47	2398 568	645	4381 993	708	·97	242 629	91	505 689	176
·48	2437 995	653	4462 436	715	·98	240 963	91	502 744	176
·49	2476 769	659	4542 164	720	0·99	239 206	89	499 623	175
0·50	0·2514 884	664	0·4621 172	728	1·00	0·0237 360	88	0·0496 327	172

m	0·3		0·4		0·5		0·6	
u	Z(u)	Δ″	Z(u)	Δ″	Z(u)	Δ″	Z(u)	Δ″
		–		–		–		–
0·50	0·0666 081	243	0·0907 938	319	0·1163 885	392	0·1438 509	463
·51	674 880	243	0920 121	320	1179 761	396	1458 477	468
·52	683 436	248	0931 984	326	1195 241	399	1477 977	471
·53	691 744	250	0943 521	327	1210 322	403	1497 006	476
·54	699 802	252	0954 731	330	1225 000	407	1515 559	480
·55	707 608	254	0965 611	334	1239 271	410	1533 632	484
·56	715 160	257	0976 157	336	1253 132	412	1551 221	486
·57	722 455	258	0986 367	339	1266 581	416	1568 324	489
·58	729 492	261	0996 238	341	1279 614	419	1584 938	494
·59	736 268	262	1005 768	344	1292 228	421	1601 058	495
·60	742 782	265	1014 954	345	1304 421	423	1616 683	498
·61	749 031	265	1023 795	347	1316 191	426	1631 810	501
·62	755 015	268	1032 289	350	1327 535	427	1646 436	502
·63	760 731	269	1040 433	350	1338 452	430	1660 560	504
·64	766 178	269	1048 227	353	1348 939	431	1674 180	507
·65	771 356	272	1055 668	354	1358 995	432	1687 293	507
·66	776 262	272	1062 755	355	1368 619	435	1699 899	509
·67	780 896	274	1069 487	356	1377 808	435	1711 996	510
·68	785 256	274	1075 863	358	1386 562	435	1723 583	510
·69	789 342	274	1081 881	357	1394 881	438	1734 660	512
·70	793 154	277	1087 542	360	1402 762	437	1745 225	512
·71	796 689	276	1092 843	358	1410 206	439	1755 278	513
·72	799 948	277	1097 786	361	1417 211	437	1764 818	512
·73	802 930	276	1102 368	360	1423 779	440	1773 846	512
·74	805 636	279	1106 590	360	1429 907	438	1782 362	513
·75	808 063	276	1110 452	361	1435 597	439	1790 365	511
·76	810 214	279	1113 953	360	1440 848	438	1797 857	512
·77	812 086	277	1117 094	361	1445 661	437	1804 837	511
·78	813 681	277	1119 874	359	1450 037	439	1811 306	509
·79	814 999	278	1122 295	360	1453 974	435	1817 266	509
·80	816 039	276	1124 356	359	1457 476	437	1822 717	507
·81	816 803	277	1126 058	358	1460 541	434	1827 661	506
·82	817 290	277	1127 402	357	1463 172	435	1832 099	506
·83	817 500	274	1128 389	358	1465 368	431	1836 031	502
·84	817 436	275	1129 018	354	1467 133	432	1839 461	501
·85	817 097	274	1129 293	356	1468 466	430	1842 390	499
·86	816 484	274	1129 212	352	1469 369	427	1844 820	498
·87	815 597	271	1128 779	353	1469 845	427	1846 752	495
·88	814 439	271	1127 993	350	1469 894	424	1848 189	492
·89	813 010	270	1126 857	349	1469 519	423	1849 134	491
·90	811 311	269	1125 372	348	1468 721	420	1849 588	487
·91	809 343	267	1123 539	345	1467 503	418	1849 555	485
·92	807 108	266	1121 361	344	1465 867	416	1849 037	482
·93	804 607	265	1118 839	341	1463 815	414	1848 037	480
·94	801 841	263	1115 976	341	1461 349	410	1846 557	475
·95	798 812	261	1112 772	337	1458 473	409	1844 602	474
·96	795 522	259	1109 231	336	1455 188	406	1842 173	470
·97	791 973	259	1105 354	333	1451 497	402	1839 274	466
·98	788 165	255	1101 144	331	1447 404	401	1835 909	463
0·99	784 102	255	1096 603	328	1442 910	396	1832 081	460
1·00	0·0779 784	251	0·1091 734	326	0·1438 020	395	0·1827 793	456

m	0·7		0·8		0·9		1·0	
u	*Z*(*u*)	Δ''	*Z*(*u*)	Δ''	*Z*(*u*)	Δ''	*Z*(*u*)	Δ''
		−		−		−		−
0·50	0·1739 739	533	0·2083 672	600	0·2514 884	664	0·4621 172	728
·51	1764 357	536	2113 821	605	2552 335	669	4699 452	732
·52	1788 439	543	2143 365	608	2589 117	675	4777 000	737
·53	1811 978	545	2172 301	615	2625 224	679	4853 811	742
·54	1834 972	551	2200 622	618	2660 652	683	4929 880	747
·55	1857 415	554	2228 325	621	2695 397	688	5005 202	750
·56	1879 304	558	2255 407	627	2729 454	692	5079 774	753
·57	1900 635	560	2281 862	629	2762 819	693	5153 593	758
·58	1921 406	565	2307 688	632	2795 491	699	5226 654	759
·59	1941 612	566	2332 882	636	2827 464	699	5298 956	762
·60	1961 252	570	2357 440	637	2858 738	703	5370 496	765
·61	1980 322	572	2381 361	639	2889 309	704	5441 271	766
·62	1998 820	574	2404 643	643	2919 176	707	5511 280	767
·63	2016 744	575	2427 282	642	2948 336	707	5580 522	768
·64	2034 093	577	2449 279	646	2976 789	710	5648 996	770
·65	2050 865	579	2470 630	646	3004 532	709	5716 700	770
·66	2067 058	580	2491 335	647	3031 566	710	5783 634	769
·67	2082 671	581	2511 393	647	3057 890	710	5849 799	770
·68	2097 703	581	2530 804	648	3083 504	712	5915 194	769
·69	2112 154	582	2549 567	649	3108 406	709	5979 820	768
·70	2126 023	582	2567 681	647	3132 599	710	6043 678	768
·71	2139 310	583	2585 148	648	3156 082	709	6106 768	765
·72	2152 014	581	2601 967	648	3178 856	708	6169 093	764
·73	2164 137	583	2618 138	645	3200 922	707	6230 654	763
·74	2175 677	580	2633 664	647	3222 281	705	6291 452	760
·75	2186 637	581	2648 543	643	3242 935	703	6351 490	758
·76	2197 016	579	2662 779	644	3262 886	702	6410 770	755
·77	2206 816	579	2676 371	640	3282 135	698	6469 295	753
·78	2216 037	576	2689 323	640	3300 686	698	6527 067	749
·79	2224 682	577	2701 635	638	3318 539	694	6584 090	745
·80	2232 750	573	2713 309	634	3335 698	691	6640 368	743
·81	2240 245	573	2724 349	634	3352 166	689	6695 903	739
·82	2247 167	569	2734 755	630	3367 945	685	6750 699	735
·83	2253 520	569	2744 531	628	3383 039	681	6804 760	730
·84	2259 304	565	2753 679	625	3397 452	680	6858 091	727
·85	2264 523	564	2762 202	621	3411 185	674	6910 695	722
·86	2269 178	561	2770 104	619	3424 244	671	6962 577	718
·87	2273 272	557	2777 387	615	3436 632	666	7013 741	712
·88	2276 809	556	2784 055	612	3448 354	664	7064 193	708
·89	2279 790	552	2790 111	609	3459 412	658	7113 937	702
·90	2282 219	549	2795 558	604	3469 812	654	7162 979	698
·91	2284 099	545	2800 401	600	3479 558	649	7211 323	693
·92	2285 434	543	2804 644	598	3488 655	645	7258 974	686
·93	2286 226	539	2808 289	592	3497 107	641	7305 939	681
·94	2286 479	536	2811 342	588	3504 918	634	7352 223	676
·95	2286 196	531	2813 807	585	3512 095	631	7397 831	670
·96	2285 382	528	2815 687	579	3518 641	625	7442 769	664
·97	2284 040	524	2816 988	575	3524 562	619	7487 043	658
·98	2282 174	520	2817 714	571	3529 864	615	7530 659	652
0·99	2279 788	517	2817 869	566	3534 551	610	7573 623	645
1·00	0·2276 885	510	0·2817 458	561	0·3538 628	603	0·7615 942	641

m	0·1		0·2		0·3		0·4	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''
1·00	0·0237 360	88	0·0496 327	172	0·0779 784	251	0·1091 734	326
·01	235 426	89	492 859	172	775 215	251	1086 539	324
·02	233 403	86	489 219	170	770 395	247	1081 020	320
·03	231 294	87	485 409	168	765 328	247	1075 181	317
·04	229 098	86	481 431	167	760 014	242	1069 025	316
·05	226 816	84	477 286	166	754 458	242	1062 553	312
·06	224 450	84	472 975	162	748 660	238	1055 769	308
·07	222 000	82	468 502	162	742 624	237	1048 677	307
·08	219 468	82	463 867	160	736 351	233	1041 278	303
·09	216 854	80	459 072	157	729 845	231	1033 576	299
·10	214 160	80	454 120	157	723 108	229	1025 575	296
·11	211 386	79	449 011	154	716 142	226	1017 278	294
·12	208 533	76	443 748	151	708 950	223	1008 687	289
·13	205 604	77	438 334	151	701 535	220	0999 807	287
·14	202 598	76	432 769	147	693 900	217	0990 640	283
·15	199 516	73	427 057	146	686 048	215	0981 190	278
·16	196 361	72	421 199	144	677 981	211	0971 462	277
·17	193 134	72	415 197	141	669 703	209	0961 457	271
·18	189 835	70	409 054	139	661 216	205	0951 181	269
·19	186 466	70	402 772	136	652 524	203	0940 636	264
·20	183 027	66	396 354	135	643 629	198	0929 827	261
·21	179 522	67	389 801	132	634 536	197	0918 757	257
·22	175 950	65	383 116	130	625 246	192	0907 430	254
·23	172 313	63	376 301	126	615 764	190	0895 849	248
·24	168 613	63	369 360	126	606 092	186	0884 020	245
·25	164 850	60	362 293	121	596 234	183	0871 946	242
·26	161 027	60	355 105	120	586 193	179	0859 630	237
·27	157 144	57	347 797	117	575 973	176	0847 077	233
·28	153 204	56	340 372	114	565 577	173	0834 291	229
·29	149 208	56	332 833	112	555 008	168	0821 276	224
·30	145 156	53	325 182	110	544 271	166	0808 037	222
·31	141 051	51	317 421	105	533 368	161	0794 576	216
·32	136 895	51	309 555	104	522 304	159	0780 899	211
·33	132 688	49	301 585	101	511 081	154	0767 011	209
·34	128 432	46	293 514	98	499 704	152	0752 914	204
·35	124 130	46	285 345	95	488 175	146	0738 613	198
·36	119 782	44	277 081	93	476 500	144	0724 114	196
·37	115 390	43	268 724	89	464 681	139	0709 419	190
·38	110 955	40	260 278	86	452 723	136	0694 534	187
·39	106 480	39	251 746	85	440 629	133	0679 462	181
·40	101 966	37	243 129	80	428 402	127	0664 209	177
·41	097 415	36	234 432	78	416 048	125	0648 779	173
·42	092 828	34	225 657	75	403 569	121	0633 176	168
·43	088 207	32	216 807	72	390 969	116	0617 405	165
·44	083 554	30	207 885	69	378 253	113	0601 469	158
·45	078 871	30	198 894	66	365 424	108	0585 375	155
·46	074 158	26	189 837	63	352 487	105	0569 126	151
·47	069 419	26	180 717	59	339 445	101	0552 726	144
·48	064 654	24	171 538	58	326 302	97	0536 182	142
·49	059 865	21	162 301	52	313 062	93	0519 496	136
1·50	0·0055 055	21	0·0153 012	52	0·0299 729	88	0·0502 674	132

m	0·5		0·6		0·7		0·8	
u	$Z(u)$	$\overline{\Delta''}$	$Z(u)$	$\overline{\Delta''}$	$Z(u)$	$\overline{\Delta''}$	$Z(u)$	$\overline{\Delta''}$
1·00	0·1438 020	395	0·1827 793	456	0·2276 885	510	0·2817 458	561
·01	1432 735	390	1823 049	452	2273 472	509	2816 486	556
·02	1427 060	387	1817 853	449	2269 550	503	2814 958	552
·03	1420 998	385	1812 208	444	2265 125	498	2812 878	546
·04	1414 551	381	1806 119	442	2260 202	495	2810 252	541
·05	1407 723	377	1799 588	436	2254 784	490	2807 085	537
·06	1400 518	374	1792 621	433	2248 876	485	2803 381	531
·07	1392 939	371	1785 221	428	2242 483	481	2799 140	525
·08	1384 989	366	1777 393	425	2235 609	476	2794 386	521
·09	1376 673	363	1769 140	420	2228 259	471	2789 105	516
·10	1367 994	358	1760 467	416	2220 438	466	2783 308	509
·11	1358 957	357	1751 378	411	2212 151	462	2777 002	505
·12	1349 563	350	1741 878	408	2203 402	456	2770 191	499
·13	1339 819	348	1731 970	402	2194 197	452	2762 881	494
·14	1329 727	343	1721 660	399	2184 540	446	2755 077	488
·15	1319 292	339	1710 951	393	2174 437	442	2746 785	482
·16	1308 518	335	1699 849	389	2163 892	437	2738 011	478
·17	1297 409	332	1688 358	384	2152 910	431	2728 759	471
·18	1285 968	326	1676 483	380	2141 497	426	2719 036	466
·19	1274 201	322	1664 228	375	2129 658	422	2708 847	462
·20	1262 112	319	1651 598	370	2117 397	416	2698 196	453
·21	1249 704	313	1638 598	366	2104 720	410	2687 092	450
·22	1236 983	310	1625 232	360	2091 633	406	2675 538	444
·23	1223 952	305	1611 506	356	2078 140	401	2663 540	438
·24	1210 616	300	1597 424	351	2064 246	395	2651 104	432
·25	1196 980	297	1582 991	346	2049 957	390	2638 236	427
·26	1183 047	291	1568 212	341	2035 278	385	2624 941	421
·27	1168 823	286	1553 092	337	2020 214	379	2611 225	416
·28	1154 313	284	1537 635	330	2004 771	375	2597 093	411
·29	1139 519	277	1521 848	327	1988 953	369	2582 530	403
·30	1124 448	273	1505 734	322	1972 766	363	2567 604	400
·31	1109 104	269	1489 298	315	1956 216	359	2552 256	393
·32	1093 491	263	1472 547	312	1939 307	353	2536 519	389
·33	1077 615	260	1455 484	306	1922 045	347	2520 391	381
·34	1061 479	254	1438 115	302	1904 436	344	2503 882	378
·35	1045 089	249	1420 444	296	1886 483	337	2486 995	371
·36	1028 450	246	1402 477	290	1868 193	332	2469 737	366
·37	1011 565	239	1384 220	288	1849 571	327	2452 113	361
·38	0994 441	236	1365 675	280	1830 622	322	2434 128	354
·39	0977 081	230	1346 850	276	1811 351	316	2415 789	351
·40	0959 491	225	1327 749	271	1791 764	311	2397 099	344
·41	0941 676	222	1308 377	266	1771 866	307	2378 065	340
·42	0923 639	215	1288 739	261	1751 661	300	2358 691	333
·43	0905 387	211	1268 840	256	1731 156	297	2338 984	329
·44	0886 924	207	1248 685	250	1710 354	289	2318 948	323
·45	0868 254	200	1228 280	247	1689 263	287	2298 589	318
·46	0849 384	197	1207 628	239	1667 885	279	2277 912	314
·47	0830 317	192	1186 737	236	1646 228	276	2256 921	307
·48	0811 058	186	1165 610	231	1624 295	269	2235 623	303
·49	0791 613	181	1144 252	225	1602 093	266	2214 022	298
1·50	0·0771 987	177	0·1122 669	221	0·1579 625	260	0·2192 123	292

m	0·9		1·0		m	0·1		0·2	
u	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	u	$Z(u)$	Δ''	$Z(u)$	Δ''
1·00	0·3538 628	603	0·7615 942	641	1·50	0·0055 055	−21	0·0153 012	−52
·01	3542 102	599	7657 620	633	·51	050 224	−18	143 671	−47
·02	3544 977	592	7698 665	627	·52	045 375	−16	134 283	−44
·03	3547 260	588	7739 083	620	·53	040 510	−15	124 851	−41
·04	3548 955	581	7778 881	615	·54	035 630	−13	115 378	−39
·05	3550 069	575	7818 064	608	·55	030 737	−12	105 866	−34
·06	3550 608	570	7856 639	602	·56	025 832	−9	096 320	−32
·07	3550 577	565	7894 612	594	·57	020 918	−7	086 742	−29
·08	3549 981	557	7931 991	589	·58	015 997	−6	077 135	−25
·09	3548 828	553	7968 781	581	·59	011 070	−5	067 503	−22
·10	3547 122	546	8004 990	575	·60	006 138	−1	057 849	−20
·11	3544 870	541	8040 624	569	·61	+ 001 205	−1	048 175	−15
·12	3542 077	534	8075 689	561	·62	− 003 729	+1	038 486	−13
·13	3538 750	528	8110 193	556	·63	008 662	+3	028 784	−9
·14	3534 895	522	8144 141	548	·64	013 592	+6	019 073	−7
·15	3530 518	516	8177 541	542	·65	018 516	+6	+ 009 355	−3
·16	3525 625	511	8210 399	535	·66	023 434	+9	− 000 366	0
·17	3520 221	504	8242 722	529	·67	028 343	+10	010 087	+5
·18	3514 313	498	8274 516	521	·68	033 242	+12	019 803	+4
·19	3507 907	492	8305 789	516	·69	038 129	+14	029 515	+11
·20	3501 009	485	8336 546	508	·70	− 043 002	+16	− 039 216	+13
·21	3493 626	481	8366 795	502	·71	047 859	+17	048 904	+16
·22	3485 762	473	8396 542	496	·72	052 699	+20	058 576	+19
·23	3477 425	468	8425 793	488	·73	057 519	+21	068 229	+23
·24	3468 620	462	8454 556	483	·74	062 318	+22	077 859	+25
·25	3459 353	456	8482 836	475	·75	− 067 095	+25	− 087 464	+29
·26	3449 630	449	8510 641	469	·76	071 847	+26	097 040	+32
·27	3439 458	444	8537 977	464	·77	076 573	+28	106 584	+36
·28	3428 842	438	8564 849	456	·78	081 271	+30	116 092	+37
·29	3417 788	432	8591 265	449	·79	085 939	+31	125 563	+42
·30	3406 302	426	8617 232	445	·80	− 090 576	+33	− 134 992	+45
·31	3394 390	420	8642 754	437	·81	095 180	+35	144 376	+47
·32	3382 058	414	8667 839	431	·82	099 749	+37	153 713	+51
·33	3369 312	409	8692 493	424	·83	104 281	+38	162 999	+54
·34	3356 157	402	8716 723	420	·84	108 775	+39	172 231	+57
·35	3342 600	398	8740 533	412	·85	− 113 230	+43	− 181 406	+60
·36	3328 645	390	8763 931	407	·86	117 642	+42	190 521	+63
·37	3314 300	387	8786 922	400	·87	122 012	+45	199 573	+66
·38	3299 568	379	8809 513	395	·88	126 337	+46	208 559	+70
·39	3284 457	374	8831 709	388	·89	130 616	+48	217 475	+71
·40	3268 972	370	8853 517	384	·90	− 134 847	+50	− 226 320	+76
·41	3253 117	362	8874 941	376	·91	139 028	+51	235 089	+77
·42	3236 900	359	8895 989	371	·92	143 158	+52	243 781	+82
·43	3220 324	352	8916 666	366	·93	147 236	+52	252 391	+84
·44	3203 396	347	8936 977	359	·94	151 260	+57	260 917	+86
·45	3186 121	342	8956 929	355	·95	− 155 227	+56	− 269 357	+90
·46	3168 504	336	8976 526	348	·96	159 138	+58	277 707	+93
·47	3150 551	332	8995 775	344	·97	162 991	+61	285 964	+96
·48	3132 266	325	9014 680	338	·98	166 783	+61	294 125	+97
·49	3113 656	322	9033 247	331	1·99	170 514	+63	302 189	+102
1·50	0·3094 724	315	0·9051 483	328	2·00	−0·0174 182	+65	−0·0310 151	+103

m	0·3		0·4		0·5		0·6	
u	*Z*(*u*)	Δ″	*Z*(*u*)	Δ″	*Z*(*u*)	Δ″	*Z*(*u*)	Δ″ −
1·50	0·0299 729	−88	0·0502 674	−132	0·0771 987	−177	0·1122 669	221
·51	286 308	−85	485 720	−127	752 184	−172	1100 865	214
·52	272 802	−80	468 639	−123	732 209	−166	1078 847	211
·53	259 216	−77	451 435	−117	712 068	−163	1056 618	206
·54	245 553	−73	434 114	−113	691 764	−159	1034 183	199
·55	231 817	−68	416 680	−109	671 304	−151	1011 549	196
·56	218 013	−64	399 137	−104	650 693	−148	0988 719	190
·57	204 145	−60	381 490	−98	629 934	−142	0965 699	185
·58	190 217	−56	363 745	−95	609 033	−136	0942 494	180
·59	176 233	−52	345 905	−89	587 996	−133	0919 109	175
·60	162 197	−47	327 976	−85	566 826	−126	0895 549	171
·61	148 114	−44	309 962	−81	545 530	−123	0871 818	164
·62	133 987	−40	291 867	−74	524 111	−116	0847 923	161
·63	119 820	−34	273 698	−72	502 576	−113	0823 867	155
·64	105 619	−32	255 457	−65	480 928	−106	0799 656	151
·65	091 386	−26	237 151	−61	459 174	−102	0775 294	145
·66	077 127	−23	218 784	−56	437 318	−98	0750 787	140
·67	062 845	−19	200 361	−52	415 362	−91	0726 140	136
·68	048 544	−13	181 886	−47	393 319	−86	0701 357	130
·69	034 230	−11	163 364	−42	371 180	−81	0676 444	126
·70	019 905	−6	144 800	−37	348 972	−76	0651 405	120
·71	+ 005 574	−1	126 199	−32	326 680	−72	0626 246	117
·72	− 008 758	+3	107 566	−28	304 316	−66	0600 970	110
·73	023 087	+6	088 905	−22	281 886	−63	0575 584	106
·74	037 410	+11	070 222	−19	259 393	−57	0550 092	102
·75	− 051 722	+15	051 520	−13	236 843	−51	0524 498	96
·76	066 019	+20	032 805	−8	214 242	−48	0498 808	91
·77	080 296	+23	+ 014 082	−4	191 593	−42	0473 037	87
·78	094 550	+28	− 004 645	+1	168 902	−37	0447 159	81
·79	108 776	+32	023 371	+6	146 174	−32	0421 210	78
·80	− 122 970	+36	− 042 091	+11	123 414	−26	0395 183	71
·81	137 128	+40	060 800	+16	100 628	−23	0369 085	67
·82	151 246	+45	079 493	+20	077 819	−17	0342 920	63
·83	165 319	+48	098 166	+25	054 993	−12	0316 692	58
·84	179 344	+53	116 814	+31	032 155	−7	0290 406	52
·85	− 193 316	+57	− 135 431	+34	+ 009 310	−3	0264 068	47
·86	207 231	+61	154 014	+39	− 013 538	+4	0237 683	44
·87	221 085	+65	172 558	+45	036 382	+8	0211 254	38
·88	234 874	+69	191 057	+49	059 218	+13	0184 787	34
·89	248 594	+74	209 507	+54	082 041	+18	0158 286	28
·90	− 262 240	+77	− 227 903	+58	− 104 846	+22	0131 757	24
·91	275 809	+82	246 241	+64	127 629	+29	0105 204	19
·92	289 296	+85	264 515	+69	150 383	+33	0078 632	14
·93	302 698	+89	282 720	+72	173 104	+37	0052 046	10
·94	316 011	+95	300 853	+78	195 788	+44	+ 0025 450	−4
·95	− 329 229	+97	− 318 908	+82	− 218 428	+47	0001 150	0
·96	342 350	+102	336 881	+87	241 021	+54	0027 750	+5
·97	355 369	+105	354 767	+93	263 560	+57	0054 345	+10
·98	368 283	+110	372 560	+96	286 042	+64	0080 930	+15
1·99	381 087	+113	390 257	+101	308 400	+67	0107 500	+19
2·00	−0·0393 778	+118	−0·0407 853	+104	−0·0330 811	+70	−0·0134 051	−24

u	0·7 $Z(u)$	Δ'' −	0·8 $Z(u)$	Δ'' −	0·9 $Z(u)$	Δ'' −	1·0 $Z(u)$	Δ'' −
1·50	0·1579 625	260	0·2192 123	292	0·3094 724	315	0·9051 483	328
·51	1556 807	255	2169 932	288	3075 477	311	9069 391	322
·52	1533 914	249	2147 453	283	3055 919	305	9086 977	317
·53	1510 682	246	2124 691	277	3036 056	302	9104 246	311
·54	1487 204	239	2101 652	274	3015 891	295	9121 204	307
·55	1463 487	236	2078 339	267	2995 431	292	9137 855	301
·56	1439 534	230	2054 759	264	2974 679	286	9154 205	297
·57	1415 351	224	2030 915	257	2953 641	281	9170 258	292
·58	1390 944	221	2006 814	255	2932 322	277	9186 019	287
·59	1366 310	216	1982 458	248	2910 726	273	9201 493	281
·60	1341 472	210	1957 854	245	2888 857	267	9216 686	279
·61	1316 218	206	1933 005	239	2866 721	263	9231 600	272
·62	1291 158	200	1907 917	237	2844 322	259	9246 242	268
·63	1265 695	197	1882 592	227	2821 664	254	9260 616	264
·64	1240 041	191	1857 040	228	2798 752	249	9274 726	260
·65	1214 193	187	1831 260	221	2775 591	246	9288 576	254
·66	1188 138	182	1805 259	217	2752 184	241	9302 172	251
·67	1161 941	177	1779 041	212	2728 536	237	9315 517	246
·68	1135 547	172	1752 611	209	2704 651	233	9328 616	243
·69	1108 981	169	1725 972	203	2680 533	229	9341 472	237
·70	1082 246	162	1699 130	200	2656 186	223	9354 091	235
·71	1055 349	160	1672 088	195	2631 616	222	9366 475	229
·72	1027 292	153	1644 851	191	2606 824	216	9378 630	226
·73	1001 082	150	1617 423	187	2581 816	212	9390 559	222
·74	0973 722	144	1589 808	182	2556 596	209	9402 266	218
·75	0946 215	141	1562 011	179	2531 167	204	9413 755	214
·76	0919 573	136	1534 035	174	2505 534	202	9425 030	211
·77	0890 792	131	1505 885	170	2479 699	197	9436 094	206
·78	0862 880	127	1477 565	166	2453 667	193	9446 952	204
·79	0834 843	122	1449 079	163	2427 442	191	9457 606	200
·80	0806 680	118	1420 430	157	2401 026	185	9468 060	195
·81	0778 401	114	1391 624	155	2374 425	184	9478 319	194
·82	0750 008	109	1362 663	151	2347 640	178	9488 384	188
·83	0721 506	104	1333 551	145	2320 677	176	9498 261	187
·84	0692 900	101	1304 294	144	2293 538	172	9507 951	181
·85	0664 193	95	1274 893	138	2266 227	170	9517 460	181
·86	0635 391	93	1245 354	135	2238 746	165	9526 788	175
·87	0606 496	86	1215 680	131	2211 100	162	9535 941	173
·88	0577 515	83	1185 875	128	2183 292	159	9544 921	170
·89	0548 451	79	1155 942	123	2155 325	155	9553 731	166
·90	0519 308	74	1125 886	121	2127 203	154	9562 375	165
·91	0490 091	70	1095 709	116	2098 927	148	9570 854	160
·92	0460 804	66	1065 416	112	2070 503	147	9579 173	158
·93	0431 451	61	1035 011	110	2041 932	144	9587 334	155
·94	0402 037	57	1004 496	105	2013 217	139	9595 340	152
·95	0372 566	53	0973 876	103	1984 363	138	9603 194	150
·96	0343 042	49	0943 153	97	1955 371	134	9610 898	146
·97	0313 469	44	0912 333	96	1926 245	132	9618 456	144
·98	0283 852	40	0881 417	91	1896 987	128	9625 870	142
1·99	0254 195	36	0850 410	88	1867 601	126	9633 142	138
2·00	0·0224 502	32	0·0819 315	84	0·1838 089	122	0·9640 276	137

m		0·7			0·8		0·9		1·0	
u		*Z*(*u*)	Δ″		*Z*(*u*)	Δ″	*Z*(*u*)	Δ″ —	*Z*(*u*)	Δ″ —
2·00		0·0224 502	− 32		0·0819 315	−84	0·1838 089	122	0·9640 276	137
·01		0194 777	− 28		788 136	−82	1808 455	121	9647 273	133
·02		0165 024	− 22		756 875	−77	1778 700	117	9654 137	132
·03		0135 249	− 20		725 537	−74	1748 828	115	9660 869	128
·04		0105 454	− 14		694 125	−71	1718 841	112	9667 473	127
·05		0075 645	− 12		662 642	−67	1688 742	109	9673 950	124
·06		0045 824	− 5		631 092	−65	1658 534	107	9680 303	122
·07	+	0015 998	− 3		599 477	−60	1628 219	105	9686 534	119
·08	−	0013 831	+ 2		567 802	−58	1597 799	101	9692 646	118
·09		0043 658	+ 6		536 069	−54	1567 278	99	9698 640	115
·10	−	0073 479	+ 11		504 282	−50	1536 658	97	9704 519	112
·11		0103 289	+ 14		472 445	−48	1505 941	94	9710 286	112
·12		0133 085	+ 19		440 560	−45	1475 130	91	9715 941	109
·13		0162 862	+ 22		408 630	−40	1444 228	91	9721 487	106
·14		0192 617	+ 28		376 660	−38	1413 235	86	9726 927	105
·15		0222 344	+ 32		344 652	−35	1382 156	85	9732 262	103
·16		0252 039	+ 35		312 609	−30	1350 992	82	9737 494	101
·17		0281 699	+ 40		280 536	−29	1319 746	80	9742 625	99
·18		0311 319	+ 44		248 434	−25	1288 420	78	9747 657	97
·19		0340 895	+ 48		216 307	−21	1257 016	76	9752 592	96
·20	−	0370 423	+ 53		184 159	−18	1225 536	72	9757 431	93
·21		0399 898	+ 56		151 993	−15	1193 984	72	9762 177	91
·22		0429 317	+ 62		119 812	−13	1162 360	69	9766 832	91
·23		0458 674	+ 65		087 618	− 8	1130 667	67	9771 396	88
·24		0487 966	+ 70		055 416	− 6	1098 907	64	9775 872	87
·25	−	0517 188	+ 73	+	023 208	− 1	1067 083	62	9780 261	85
·26		0546 337	+ 79	−	009 001	0	1035 197	61	9784 565	83
·27		0575 407	+ 83		041 210	+ 4	1003 250	58	9788 786	82
·28		0604 394	+ 86		073 415	+ 7	0971 245	57	9792 925	80
·29		0633 295	+ 92		105 613	+11	0939 183	53	9796 984	79
·30	−	0662 104	+ 95	−	137 800	+13	0907 068	53	9800 964	77
·31		0690 818	+100		169 974	+18	0874 900	50	9804 867	76
·32		0719 432	+104		202 130	+19	0842 682	48	9808 694	75
·33		0747 942	+110		234 267	+24	0810 416	46	9812 446	72
·34		0776 342	+112		266 380	+27	0778 104	44	9816 126	72
·35	−	0804 630	+118	−	298 466	+29	0745 748	43	9819 734	70
·36		0832 800	+122		330 523	+34	0713 349	39	9823 272	69
·37		0860 848	+127		362 546	+35	0680 911	39	9826 741	67
·38		0888 769	+130		394 534	+40	0648 434	37	9830 143	67
·39		0916 560	+136		426 482	+43	0615 920	34	9833 478	64
·40	−	0944 215	+141	−	458 387	+46	0583 372	32	9836 749	65
·41		0971 729	+143		490 246	+50	0550 792	32	9839 955	61
·42		0999 100	+150		522 055	+52	0518 180	28	9843 100	63
·43		1026 321	+153		553 812	+56	0485 540	27	9846 182	59
·44		1053 389	+159		585 513	+59	0452 873	25	9849 205	59
·45	−	1080 298	+163	−	617 155	+63	0420 181	23	9852 169	58
·46		1107 044	+167		648 734	+66	0387 466	22	9855 075	56
·47		1133 623	+172		680 247	+69	0354 729	20	9857 925	57
·48		1160 030	+177		711 691	+73	0321 972	17	9860 718	54
·49		1186 260	+182		743 062	+76	0289 198	16	9863 457	53
2·50		−0·1212 308	+187		−0·0774 357	+79	0·0256 408	14	0·9866 143	53

m	0·9		1·0	
u	Z(u)	Δ″	Z(u)	Δ″ −
2·50	0·0256 408	−14	0·9866 143	53
·51	0223 604	−12	9868 776	51
·52	0190 788	−11	9871 358	51
·53	0157 961	− 9	9873 889	49
·54	0125 125	− 7	9876 371	49
·55	0092 282	− 4	9878 804	47
·56	0059 435	− 4	9881 190	47
·57	+ 0026 584	− 1	9883 529	46
·58	− 0006 268	0	9885 822	45
·59	0039 120	+ 2	9888 070	44
·60	− 0071 970	+ 4	9890 274	43
·61	0104 816	+ 6	9892 435	42
·62	0137 656	+ 8	9894 554	42
·63	0170 488	+ 9	9896 631	41
·64	0203 311	+11	9898 667	39
·65	− 0236 123	+13	9900 664	40
·66	0268 922	+14	9902 621	38
·67	0301 707	+18	9904 540	37
·68	0334 474	+18	9906 422	38
·69	0367 223	+20	9908 266	35
·70	− 0399 952	+22	9910 075	37
·71	0432 659	+24	9911 847	34
·72	0465 342	+26	9913 585	34
·73	0497 999	+28	9915 289	33
·74	0530 628	+29	9916 960	34
2·75	− 0·0563 228	+32	0·9918 597	31

m	0·9		1·0	
u	Z(u)	Δ″	Z(u)	Δ″ −
2·75	− 0·0563 228	+32	0·9918 597	31
·76	0595 796	+33	9920 203	32
·77	0628 331	+35	9921 777	31
·78	0660 831	+37	9923 320	31
·79	0693 294	+40	9924 832	29
·80	− 0725 717	+40	9926 315	29
·81	0758 100	+44	9927 769	29
·82	0790 439	+44	9929 194	28
·83	0822 734	+47	9930 591	27
·84	0854 982	+49	9931 961	27
·85	− 0887 181	+51	9933 304	27
·86	0919 329	+53	9934 620	25
·87	0951 424	+55	9935 911	26
·88	0983 464	+56	9937 176	25
·89	1015 448	+60	9938 416	24
·90	− 1047 372	+61	9939 632	24
·91	1079 235	+64	9940 824	24
·92	1111 034	+64	9941 992	23
·93	1142 769	+69	9943 137	22
·94	1174 435	+68	9944 260	22
·95	− 1206 033	+73	9945 361	22
·96	1237 558	+74	9946 440	21
·97	1269 009	+77	9947 498	22
·98	1300 383	+78	9948 534	19
2·99	1331 679	+81	9949 551	20
3·00	−0·1362 894	+83	0·9950 548	21

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